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ACCOUNT OF THE

LIFE AND WRITINGS

OF

ROBERT SIMSON, M.D.

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ACCOUNT OF THE

LIFE AND WRITINGS

OF

ROBERT SIMSON, M. D.

LATE PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

By the Rev. WILLIAM TRAIL, LL. D. F.R.S. Edin.
MEMBER OF THE ROYAL IRISH ACADEMY, AND CHANCELLOR OF ST. SAVIOUR'S, CONNOR.

Robertus Simson. M.D.

GEOMETRIAM SUB TYRANNO BARBARO SÆVA SERVITUTE DIU SQUALENTEM,
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THIS ACCOUNT OF THE LIFE

AND WRITINGS

OF A DISTINGUISHED MEMBER

OF THEIR SOCIETY

IS MOST RESPECTFULLY INSCRIBED.

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ADVERTISEMENT.

ABOVE thirty years ago the late EARL STANHOPE honoured me with a requeſt to draw up an account of the Life and Writings of the late Dr. SIMSON, of Glaſgow, which might be publiſhed in the new edition of the *Biographia Britannica*. The ſlow progreſs of that great work left me much at liberty as to the time of preparing an article which could appear only near the end of it; and for a number of years having been occupied by engagements of a different kind, I was in ſome meaſure compelled to poſtpone the execution of my undertaking, much longer than I wiſhed to have done.

As there is not at preſent any near proſpect of the completion of the *Biographia*, I could not properly, at my time of life, defer any longer embracing the opportunity afforded me of paying this ſmall tribute of reſpect to the memory of that eminent man, by whoſe friendſhip and inſtruction I was honoured during ſome of the laſt years of his life. I thought it my duty therefore, though the undertaking was ſtill liable to interruptions from other concerns, to collect the neceſſary materials, and to arrange them for a ſeparate publication.

With reſpect to the incidents of Dr. Simson's life, particularly thoſe of the early part of it, I obtained ſatisfactory information from a ſhort narrative drawn up by the Doctor's colleague and particular friend the late Mr. Clow, which, from the intimacy ſubſiſting between them for many years, may be conſidered as authentic and accurate. From ſeveral gentlemen who had been in the ſame College with Doctor Simson other circumſtances have been communicated, and even from my own acquaintance with him, though only for a few years, ſome intereſting particulars came within the reach of my own obſervation.

With regard to Doctor Simson's ſcientific purſuits, there are abundant materials of information in his Works which have been publiſhed, and particularly in his learned Prefaces and Notes which accompany them. Some circumſtances, alſo, have been collected from his unpubliſhed MS. papers, and from the ſmall remains of his Correſpondence. It is much to be regretted that the greater part of his Mathematical Correſpondence, which appears to have been very extenſive during a great part of his long life, had been either loſt or deſtroyed in his own time. Some intereſting fragments of it, however, were found among his papers; and a ſeries of his letters to that eminent mathematician the late Earl Stanhope, from 1750 to 1758, was in the moſt liberal and obliging manner communicated by the preſent Earl; and from them ſome curious notices of Dr. Simson's ſtudies have been extracted.

Some hiſtory and explanation of the ancient geometrical Analyſis, almoſt neceſſarily, forms a part of an account of the literary life of Dr. SIMSON, by whom that Analyſis has been completely reſtored and illuſtrated. The *Mathematical Collections* of PAPPUS, in which is contained nearly every thing that is known from the Ancients of that celebrated inſtrument of inveſtigation, which they employed ſo ſucceſsfully in their geometrical inquiries, from the Doctor's early partiality for that branch of ſcience, naturally became an object of his particular ſtudy. Some account therefore of that work, and of Dr. SIMSON's commentaries on it, is requiſite for giving a juſt view of his geometrical labours, and for eſtimating their importance to, what is ſo much wanted, a corrected edition of that valuable Author. Even with the riſk of ſeveral repetitions, it has appeared convenient to detach from the Memoir the particular account of PAPPUS, and ſome explanations of the ancient Analyſis connected with it, and to annex them as an Appendix. Theſe details, though unintereſting to many readers, may be acceptable to others; and my object in preparing them will be attained, if they can ſave ſome trouble to thoſe who at any future time may undertake a new edition of the *Mathematical Collections*. Two ſhort paſſages of PAPPUS, concerning the ancient claſſification of geometrical lines, are added in a ſecond appendix, in the original language, with COMMANDINE's verſion of them.

To theſe is ſubjoined Dr. SIMSON's tranſlation of the deſcription of the Poriſms of EUCLID, in the Preface to the ſeventh book of PAPPUS; in which are ſome material improvements of the former tranſlations.

Auguſt, 1812.

SECTION I.

General Account of DR. SIMSON's *Life.*

DOCTOR ROBERT SIMSON, late Profeſſor of Mathematics in the Univerſity of Glaſgow, was the eldeſt ſon of JOHN SIMSON, of Kirktonhill in Ayrſhire, and was born on the fourteenth of October 1687, O. S. Being deſigned by his father for the Church, after having got the uſual ſchool education, he was ſent to the Univerſity of Glaſgow, where he was diſtinguiſhed by his proficiency in claſſical learning, and in the ſciences.

At this time, from temporary circumſtances, it happened, that no Mathematical Lectures were given in the College; but young SIMSON's inquiſitive mind, from ſome fortunate incident, having been directed to Geometry, he ſoon perceived the study of that ſcience to be congenial to his taſte and capacity. This taſte however, from an apprehenſion that it might obstruct his application to ſubjects more connected with the ſtudy of theology, was anxiouſly diſcouraged by his father, though it would ſeem, with little effect.

Having procured a copy of EUCLID's Elements, with the aid only of a few preliminary explanations from ſome more advanced ſtudents, he entered on the ſtudy of that oldeſt and beſt introduction to Mathematics. In a ſhort time he read and underſtood the firſt ſix with the eleventh and twelfth books; and being delighted with the ſimplicity of language and accuracy of reaſoning in EUCLID, notwithſtanding the diſcouragements he met with, he perſevered in his Mathematical purſuits;* and by his progreſs in the more difficult branches he laid the foundation of his future eminence. But though the bent of his inclination for theſe pursuits was ſtrong, he did not neglect the other ſciences then taught in the College; and in proceeding through the regular courſe of academical ſtudy he acquired the principles of that variety of knowledge, which he retained through life, and which contributed much to the eſtimation of his converſation and manners in ſociety. His chief attention, however, was directed to his favourite ſcience; and his reputation as a mathematician in a few years became ſo high, and his general character ſo much reſpected, that in 1710, when he was only twenty-two years of age, the Members of the College, without any ſolicitation on his part, made him an offer of the Mathematical Chair, in which a vacancy was in a ſhort time expected to take place. From his natural modeſty, however, he felt much reluctance, at ſo early an age, to advance abruptly from the ſituation of a Student to that of a Profeſſor in the ſame College; and therefore he ſolicited permiſſion to ſpend one year at leaſt in London, where, beſides other obvious advantages, he might have opportunities of becoming acquainted with ſome of the eminent mathematicians of England, who were then the moſt diſtinguiſhed in Europe. In this proper requeſt he was readily indulged; and without delay he proceeded to London, where he remained about a year, diligently employed in the improvement of his mathematical knowledge.

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* In ſome of his early MS. volumes ſtill preſerved, are notices of the books he had been reading, and of which he had been making abſtracts. In one of theſe, dated in 1705, (then in his eighteenth year,) are mentioned OUGHTRED's Clavis, RAPHSON's Tracts, JONES's Synopſis, KERSEY's Algebra, and ſeveral others.

This journey turned out very favourable to his views; and he had much ſatisfaction in the acquaintance of ſome reſpectable mathematicians, particularly of Mr. JONES, Mr. CASWELL, and Mr. DITTON. With the latter, indeed, who was then Mathematical Maſter of Chriſt's-Hoſpital, and well eſteemed for his learning, he was more particularly connected; but from all of them he had opportunities of receiving information reſpecting the progreſs of the ſcience, both in England and on the continent of Europe. When the vacancy in the Profeſſorſhip of Mathematics at Glaſgow did occur in the following year, the Univerſity, while Mr. SIMSON was ſtill in London, appointed him to fill it; and the Minute of Election, which is dated March 11th, 1711, concluded with this very proper condition: " That they will admit the ſaid Mr. ROBERT " SIMSON, providing always, that he give ſatisfactory proof of " his ſkill in mathematics previous to his admiſſion." He returned to Glaſgow before the enſuing ſeſſion of the College, and having gone through the form of a trial, by reſolving a geometrical problem propoſed to him, and alſo by giving " a " ſatisfactory ſpecimen of his ſkill in Mathematics, and dex- " terity in teaching Geometry and Algebra;" having produced alſo reſpectable certificates of his knowledge of the ſcience from Mr. CASWELL, and others, he was duly admitted Profeſſor of Mathematics, on the 20th of November of that year.

Mr. SIMSON immediately after his admiſſion entered on the duties of his office, and his firſt occupation neceſſarily was the arrangement of a proper courſe of inſtruction for the ſtudents who attended his lectures, in two diſtinct claſſes. He prepared elementary ſketches of ſome branches in which there were not ſuitable treatiſes in general uſe. Among his papers one of theſe ſtill remains, a tranſlation of the three firſt books of L'HOSPITAL's Conic Sections, in which geometrical demonſtrations are ſubſtituted for the algebraical of the original, according to Mr. SIMSON's early taſte on this ſubject. There remain alſo ſome traces of his collections of Problems from HUYGHENS, GREGORY, and others, in optics and aſtronomy, for the uſe of his ſcholars; and it appears likewiſe that ſoon after his admiſſion, he had given ſome public Lectures on the Hiſtory of Mathematics, before the whole Univerſity, of which ſeveral happen to be preſerved, and are proofs of the extent and accuracy of his learning at the early period of life when they were delivered.

Both from a ſenſe of duty and from inclination, he now directed the whole of his attention to the ſtudy of mathematics; and though he had a decided preference for geometry, which continued through life, yet he did not devote himſelf to it, to the excluſion of the other branches of mathematical ſcience; and in the progreſs of this Memoir it will appear that he was well acquainted with the modern analyſis, particularly as it ſtood in the early part of his time. From 1711, he continued near fifty years to teach mathematics to two ſeparate claſſes, at different hours, five days in the week, during a continued ſeſſion (or term) of ſeven months; beſides giving occaſional inſtruction which he was ever ready to communicate to thoſe ſtudents, who wiſhed for more particular explanations of his lectures, or to make further progreſs in the ſtudy of mathematics. Though the duties of a profeſſor ſoon became familiar and eaſy to him, yet they occupied a conſiderable portion of his time, and divided it, ſo as often to interrupt the courſe of his private ſtudies.

His manner of teaching was uncommonly clear, and engaging to young people; and moſt of his ſcholars retained through life an affection and reverence for the Profeſſor. The College of Glaſgow in his time was in great repute both at home and abroad, to which Dr. HUTCHESON, Dr. MOOR, Mr. ADAM SMITH, and himſelf, much contributed. The reſort of ſtudents was great, and almoſt all of them attended Dr. SIMSON's lectures. The knowledge of the elementary branches of mathematics, and of the moſt uſeful applications of them, were thence much diffuſed in the College, and ſome taſte alſo for the ſtudy of the higher branches was excited; but the early age of the greater number of the ſtudents, their ſhort reſidence in College, and the neceſſary appropriation of a conſiderable portion of their time to other ſciences, ſeldom admitted of that long and nearly excluſive cultivation of one particular ſcience, by which alone, eſpecially in mathematics, eminence uſually can be attained. Among Mr. SIMSON's ſcholars, however, ſeveral roſe to diſtinction as mathematicians. Dr. MATTHEW STEWART, who alone has applied the geometrical method of reaſoning to the moſt complex phyſical inveſtigations, by univerſal acknowledgement, is to be named the firſt. Mr.

WILLIAMSON,[*] a favourite pupil, from whom he had great expectations, died very young. Dr. JAMES MOOR, Greek Profeſſor at Glaſgow, and Profeſſor ROBISON,[†] of Edinburgh, were all well known as mathematicians of ſuperior abilities and attainments.

In the year 1758, Dr. SIMSON, being then ſeventy-one years of age, found it neceſſary to employ an aſſiſtant in teaching; and in 1761, on his recommendation, the Rev. Dr. WILLIAMSON was appointed his aſſiſtant and ſucceſſor.

The reſignation of Dr. SIMSON preſented an opportunity to the Principal and Profeſſors of recording in their minutes the affection which they felt for the Doctor, and their high admiration of his genius. A long paper for this purpoſe was drawn up by his colleagues, and it is expreſſed with all the warmth of attachment and reſpect, which it was natural for them to entertain for the father of the College, from whom the Univerſity had derived ſo much honour. Several of them had been his pupils, and all had lived with him in habits of friendſhip from the time of their becoming members of the Society. It is introduced in the following manner:

"The Univerſity Meeting do hereby gratefully, unanimouſly, " and warmly expreſs to Dr. SIMSON their moſt cordial thanks

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[*] Mr. WILLIAMSON, afterwards the Rev. Dr. WILLIAMSON, Chaplain to the Britiſh Factory at Liſbon, in which ſtation he died.

[†] In the third and fourth editions of the Encyclopedia Britannica (in the article SIMSON R.) is a ſketch of Dr. SIMSON's life, which is known to be from the pen of Profeſſor ROBISON, of Edinburgh; and though, from ſome accidents not now to be aſcertained, there are ſome errors in dates, and in ſome ſmall circumſtances of the narrative, yet it bears the character of the diſtinguiſhed ability and knowledge of the writer.

" for his long, faithful, and eminent ſervices to the Univerſity,
" in the courſe of fifty years; during which very uncommon
" period, he has with univerſal applauſe, been Profeſſor of
" Mathematics here; to the great honour of this Univerſity,
" as well as to his own high reputation, which will laſt as long
" as united elegance and ſcience are admired among mankind."

It proceeds to mention generally ſome of his moſt diſtinguiſhed inventions, juſtly ſtating that he was the firſt, in modern times, who fully underſtood and explained the ancient analytical geometry; that he had already reſtored, with ſuperior accuracy and elegance, ſome of the moſt valuable works of the ancient geometers, which for ages had been loſt; and that he purpoſed to employ the remaining leiſure of his life, in compleating and publiſhing others, not leſs neceſſary for the full illuſtration of the nature and uſe of the ancient analyſis: " for the accompliſhment of which valuable deſign, (the Minute concludes,) " of ſo much importance to the advancement of true ſcience, the Univerſity Meeting do wiſh and " pray that he may long enjoy the bleſſing of a vigorous old " age; and they intreat Dr. Simson to be entirely aſſured, " that they will at all times heartily embrace every opportunity of teſtifying to him their gratitude, their affection, " eſteem, and veneration."

During the remaining ten years of his life, he enjoyed a pretty equal ſtate of good health; and continued to occupy himſelf in correcting and arranging ſome of his mathematical papers, and occaſionally for amuſement, in the ſolution of problems, and demonſtration of theorems, which occurred from his own ſtudies, or from the ſuggeſtions of others. His converſation on mathematical and other ſubjects continued to be clear and accurate, yet he had ſome ſtrong impreſſions of the decline of his memory, of which he frequently complained; and this probably protracted, and finally prevented his undertaking the publication of ſome of his works, which were in ſo advanced a ſtate, that with little trouble they might have been compleated for the preſs.* His only publication, however, after the reſignation of his office, was a new and improved edition of the Data of Euclid, which in 1762 was annexed to the ſecond and corrected edition of the Elements. But from that period, though much ſolicited to bring forward ſome of his other works on the ancient geometry, though he knew well how much it was deſired, and though he was fully apprized of the univerſal curioſity excited reſpecting his diſcovery and illuſtration of the Poriſms of Euclid, he reſiſted every importunity on the ſubject.†

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* The following extracts from Dr. Simson's Letters to the Writer of this Memoir mention the impreſſion he had of the decline of his memory:

In a letter dated February 11th, 1766.——"As to the publiſhing any thing on the "ancient geometry, give nobody any hopes of my doing it. You know my inability "to do any thing that requires ſo much thought and application as that would do; "and if it be not by your aſſiſtance in preparing them for the preſs, and in taking "care of the printing, I have ſcarce any hopes of doing any thing that way, though I "much deſire it." And in another letter of March 7th, 1767, he ſays, "I wiſh you "had not mentioned any thing that might give expectation of the Poriſms from me, as "it is very doubtful whether I ſhall be able to publiſh any thing either about them, "or any other geometrical ſubject; my memory, and conſequently any other ability "for ſuch things, being ſo greatly decayed. However, your kind offer of aſſiſtance "will make me give out ſome figures to be cut, though I believe they cannot be "ready againſt the time you propoſe to be here."

I ſhall have afterwards occaſion to mention the impreſſion of a decline of memory which Dr. Simson ſeems to have felt many years before.

† Sometimes he ſeems to have entertained ſerious thoughts of publiſhing the moſt important of his remaining works, the Sectio Determinata, and the Poriſms; and in

A life like Dr. Simson's, purely academical and perfectly uniform, rarely contains occurrences, the recording of which could be either intereſting or uſeful. But his mathematical labours and inventions form the important part of his life, and ſupply the beſt illuſtration of his character; and with reſpect to them, there are abundant materials of information in his printed works; and ſome circumſtances alſo may be gathered from a number of MS. papers which he left; and which, by the direction of his executor, are depoſited in the Library of the College of Glaſgow.* It is to be regretted, that of the extenſive correſpondence which he carried on through life with many diſtinguiſhed Mathematicians, a ſmall portion only is preſerved. The greater part of it had been loſt or deſtroyed in his own time; ſo that in 1751, when he was deſirous of reviewing ſome of his early letters to Dr. Jurin on a particular ſubject, he obtained a copy, probably from the archives of the Royal Society, of which Dr. Jurin at the time of the

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one of his MS. books, of which the earlieſt date is of 1762, there is the following notice. " The title-page, if the book to which it belongs come to be printed, may " be thus:

" Apollonii Pergæi
" De Sectione Determinata Libri duo,
" Reſtituti."
" Quibus adjecta ſunt non pauca Poriſmata; inter quæ habentur quædam Euclidis,
" quibus Doctrina Poriſmatum explicata et reſtituta eſt."

* By the liberal favour of the Principal and Profeſſors, the Writer of this Memoir has had every accommodation for conſulting theſe Papers, and alſo the very valuable Mathematical Library which Dr. Simson bequeathed to the Univerſity. He has received alſo from them ſome material communications reſpecting their diſtinguiſhed Colleague; and he is particularly obliged to Profeſſors Jardine, Young, and Millar, for procuring uſeful information, and aiding his reſearches.

correſpondence (1723) was ſecretary. Through Dr. JURIN he had ſome intercourſe with Dr. HALLEY; and both at that time, and afterwards, he had frequent correſpondence with Mr. MACLAURIN, with Mr. JAMES STIRLING, Dr. JAMES MOOR, Mr. WILLIAMSON of Liſbon, and more particularly with the Rev. Dr. MATTHEW STEWART; of which there are ſome notices in his printed works, and of which alſo there are ſome remains among his unpubliſhed papers. In the latter part of his life, his mathematical correſpondence was chiefly with that eminent Geometer the late Earl STANHOPE,* and with GEORGE LEWIS SCOTT, eſq; then a Commiſſioner of Exciſe, and well known for his ſcientific attainments; and from Dr. SIMSON's letters to both theſe gentlemen, ſome illuſtration of his opinions and purſuits will be obtained, which ſhall be taken notice of in a ſubſequent part of this Memoir.

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* The Doctor's affectionate reſpect for that venerable Nobleman is ſtrongly marked by the concern which he expreſſes in a letter to himſelf on hearing a report of his being indiſpoſed. "May it pleaſe the gracious GOD to preſerve ſo valuable a "life for many good purpoſes, both with reſpect to your Lordſhip's honourable family, "and the public good; and particularly for promoting of real and uſeful knowledge "of every kind."——Letter dated 7th March 1755.

SECTION II.

Particular Account of Dr. SIMSON's *Mathematical Studies, and of the Works publiſhed by himſelf.*

IT has already been obſerved, that Dr. SIMSON directed his early attention to the ancient Geometry. Dr. HALLEY, by his edition of the Conics of APOLLONIUS, and more particularly by his publication of two Treatiſes of APOLLONIUS, accompanied with a corrected edition of the preface to the ſeventh Book of PAPPUS, gave conſiderable aid to thoſe who embarked in that ſtudy. The *Mathematical Collections* of PAPPUS are indeed the chief repoſitory of information reſpecting the Geometry of the Ancients, and eſpecially reſpecting their analyſis, which has been the ſubject of much diſcuſſion among the moderns. This moſt intereſting work contains ſome curious mathematical hiſtory of former times, but is more particularly valuable, by the account, contained in the preface to the ſeventh Book, of the treatiſes of the analytical geometry of the ancients, which together obtained the name of *τόπος ἀναλυομένος*. In this preface there is, firſt, a general expoſition of the analyſis employed by the ancients, both in the ſolution of problems, and in the demonſtration of theorems; then follows a particular deſcription of the nature and contents of a certain number of theſe treatiſes, which we may preſume were conſidered by Pappus as the moſt important; and an enumeration of the whole is added, conſiſting of thirty-three books. The ſeventh book of Pappus itſelf conſiſts of a number of Lemmata, or ſubſidiary propoſitions, not contained in Euclid, but aſſumed or employed in the ſeveral treatiſes which are ſo fully deſcribed in the preface.

It is through the medium of Commandine's tranſlation only, that the importance of this work of Pappus is yet known to the public. Though many manuſcript copies of the original remain in various libraries of Europe, it has never been printed; and no attempt has been made to correct the tranſlation of Commandine, whoſe zeal, learning, and ability, in promoting the knowledge of the ancient Geometricians by uſeful tranſlations, are very meritorious; though this poſthumous verſion of Pappus, no doubt from its not having received his final corrections, remains in a leſs perfect ſtate.*

Dr. Simson, however, ſeriouſly applied himſelf to the ſtudy of this tranſlation; and notwithſtanding its unfiniſhed ſtate, it was the means of enabling him to inveſtigate, and fully to illuſtrate the principles, of the analytical geometry of the ancients. The generality of modern mathematicians, from not having fully conſidered the intimations of Pappus on this ſubject, fell into ſtrange miſapprehenſions reſpecting it.† When they examined the works of the ancient geometers,

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* For a particular account of Pappus, and of this tranſlation, ſee the Appendix.

† Vieta muſt be excepted; who, though he did not uſe the ancient analyſis in his *Apollonius Gallus,* yet he ſeems perſuaded, that the ancients had a true geometrical analyſis. For in his firſt Appendix to that Treatiſe (p. 339, Op. Vietæ) he obſerves, "At Algebra quam tradidere, Theon, Apollonius, Pappus, et alii veteres ana-" lyſtæ omnino geometrica eſt;" &c.

Euclid, Archimedes, and Apollonius, they admired the elegance and clearneſs of their demonſtrations, but wondered by what contrivance they had invented and proved ſo many curious theorems, and obtained ſolutions of ſo many difficult problems. They aſſerted even that the ancients muſt have poſſeſſed an analyſis equivalent to the algebraical; but that they had induſtriouſly concealed it, in order to excite the greater admiration of their inventions. This opinion, naturally improbable, is however unreſervedly expreſſed by ſome eminent mathematicians; by Francis Schooten, by his brother Peter Schooten, by Peter Nonius or Nunez; and what is more remarkable, is avowed in later times by the profound Dr. Wallis,* who was certainly acquainted with Pappus, both in the original, and in Commandine's tranſlation. But the important communications of that author reſpecting the geometrical analyſis of the ancients ſeem not to have been duly conſidered by this very eminent Mathematician, though he publiſhed, with learned and valuable notes, a fragment of Pappus, not contained in Commandine's tranſlation, but which he diſcovered in one of the two Savilian MSS. of the *Mathematical Collections.*

Dr. Barrow alſo, in his Lectures,§ gives countenance to this erroneous judgment on the ancients, which has been

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[*] Wallis's Algebra, chap. ii.

[§] In Dr. Barrow's four Lectures on the Diſcoveries of Archimedes, after ſtating his plan in p. 341, he adds, "unde patebit qualem analyſin, et quam noſtræ modernæ "ſimilem exercuerit." And in p. 376, after giving an algebraical analyſis of a problem, (viz. Prop. v. Lib. 2. Archimed. *de Sphæra et Cylindro,*) producing a cubic equation, he derives from it the proportion given by Archimedes, in which a certain ſtraight line is to be cut in order to reſolve the problem; he adds, "Qui ipſiſſipronounced by ſo many mathematicians of modern times; and even ſo late as the publication of Sir ISAAC NEWTON'S Fluxions in 1736, Mr. COLSON, the commentator on that work, ſtates this to be the opinion of "many of our modern geo-"metricians."‡

This charge, however, againſt the ancients, as is juſtly obſerved by Dr. SIMSON,§ was altogether groundleſs. The only algebra known to the ancients was that of DIOPHANTUS,

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"mus eſt analogiſmus iſte, ad quem rem deduxit ARCHIMEDES; quod ipſum ſatis "prodit et arguit, qualem is analyſin uſurparit. Nam huc eum deveniſſe, varias iſtas "proportionum compoſitiones, diviſiones, permutationes, ac inverſiones, quales in diſ-"curſu ſuo oſtentat adhibendo, pene ſupra fidem eſt." But in this caſe the ancient analyſis is properly applicable; and under the management of ARCHIMEDES, its power in this propoſition need not excite ſurprize. He refers alſo to the elegant ſynthetic ſolution of this problem by HUYGHENS, by means of the triſection of an arch. HUYG. Illuſtr. Probl. prob. 1. Perhaps ſome of the diſcoveries of ARCHIMEDES may have been ſuggeſted to him by views and reaſonings reſembling thoſe of CAVALLERIUS, as is conjectured by his Commentator TORELLI; but he eſtabliſhed their truth by rigorous demonſtrations, which might be naturally derived from the only analyſis of ancient times, the geometrical.

‡ See COLSON'S Commentary on NEWTON'S Fluxions, p. 143: and it may be here remarked, that the obſervations of Sir ISAAC NEWTON, (page 1,) to which this part of the Commentary refers, ſeem to imply, that he conſidered the ancient geometry, which he greatly admired, as ſynthetical only, without having much conſidered the nature and merits of their analyſis. Some of his obſervations in the *Arithmetica Universalis* naturally lead to the ſame inference. See ſect. iv. cap. 1. art. 18: alſo, Appendix *de Æquationum Constructione Lineari*, art. 51. Dr. PEMBERTON'S account of Sir ISAAC NEWTON'S opinion of the ancient geometry, of his regret that he had not ſtudied it more particularly, and alſo of his remarks on *De Omerique*, correſponds with theſe obſervations in Sir ISAAC'S works. It may be added, that opinions like that of Mr. COLSON are to be found in many reſpectable recent writers; and conſiſtently with them, the ancient geometry is generally diſtinguiſhed as *synthetical*, while the term *analysis* is almoſt excluſively applied to the modern ſyſtem.

§ See the Preface to Dr. SIMSON'S reſtoration of the *Loci Plani* of APOLLONIUS, where this opinion is particularly ſtated, with the Doctor's ſatisfactory refutation of it.

which was never applied to geometry. But they had an analyſis of their own, of which they made no ſecret, and of which there are ſome ſhort ſpecimens, even in Euclid's Elements.* This analyſis however is more fully deſcribed by Pappus, who alſo obſerves, that in order to facilitate the ſolution of geometrical problems by this method, the ancients compoſed no leſs than thirty-three books, which collected together got the title of *τόϖος ἀναλυομένος*; and of the chief of which, as has already been mentioned, he gives an intereſting account in the preface to his ſeventh book.

What led probably to this prevailing miſtake of modern mathematicians was, that in the moſt valuable ancient treatiſes ſtill remaining, ſuch as thoſe of Archimedes, and even the Conics of Apollonius, the analyſis of their propoſitions (particularly of their theorems) is generally omitted; though there can be no doubt that this method was employed by them, both in reſolving problems, and for aſcertaining the truth of theorems, which now appear in their works only in the ſynthetic form, and which had been either propoſed to them by others, or had occurred to them in their own ſtudies. In copying their books, however, for general uſe, for the ſake of ſhortneſs, (which, before the invention of printing, was an object of conſequence,) and even for facility to the reader in acquiring the knowledge of their diſcoveries, the analyſis was often omitted. In the fifth book of Pappus is an example of this omiſſion, for which he aſſigns the reaſon juſt now mentioned, and which we may believe influenced the other more ancient geometers. He obſerves, that in the compariſon which he is about to give of the five regular ſolids having equal ſurfaces, for the ſake of *brevity* and *perſpicuity*, he is to employ only the ſynthetic method, and not the analytic, which ſome of the ancients had uſed in treating that ſubject.† But in the other analytical treatiſes of APOLLONIUS, containing ſolutions of general problems for facilitating the reſolution of any particular geometrical problem which can be reduced to a caſe of them, a full analyſis of every caſe is given, being eſſential to this important application of ſuch problems. In the *Sectio Rationis*, recovered from the Arabic by Dr. HALLEY, is a ſpecimen of this compleat manner of ſolution, which, we muſt preſume, was followed in the other treatiſes, now unfortunately loſt.

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* Euclid's Elem. book xiii. prop. 1, 2, 3, 4, 5.

Theſe books appear to have been all exiſting in the time of PAPPUS, but the greater part of them have ſince periſhed, or at leaſt they are not known to remain, either in the original, or in any tranſlation. The book of *Data* by EUCLID, in the Greek; two books of *Sectio Rationis*, by APOLLONIUS, in an Arabic verſion; and ſeven books of the *Conics* of APOLLONIUS, four in Greek and three in Arabic; are all that have been preſerved. But fortunately the deſcriptions by PAPPUS of eleven more of theſe books are ſo particular and entire, that ſome eminent modern mathematicians have been able to reſtore them, with various ſucceſs indeed, as ſhall be afterwards more particularly ſtated. Thoſe by Dr. HALLEY, from his ſuperior taſte and knowledge of the ſubject, are in the pureſt ſtyle of geometry.

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† His expreſſion is, "*Καὶ τὴν ἔφοδον των ἀποδειξεῶν ἐχουσας, οὐ διὰ τῆς ἀναλυτικῆς λεγομενῆς* "*θεωρίας δὶ ἧς ἔνιοι τῶν παλάιων ἐποιοῦντο τὰς ἀποδέιξεις τῶν προειρημένων σχημάτων, ἀλλὰ διὰ* "*τῆς κατὰ συνθέσιν ἀγωγῆς ἐπὶ τὸ σαφέστερον καὶ συντομώτερον ὑπ' ἐμοῦ, διασκευάσμενας.*" &c. Ex Cod. BULLIALDI, ad fol. 99. a. COM.

The others, by VIETA, SNELLIUS, MARINUS GHETALDUS, FERMAT, SCHOOTEN, and WALLIS, of which ſome are algebraical, and ſome geometrical, though reſpectable works of ingenious men, are defective in that elegance and compleatneſs of ſolution which diſtinguiſh the analytical writings of the ancient Geometers. A field was thus ſtill open, for a perſon of Dr. SIMSON's taſte and genius, for attempting a more perfect reſtoration of theſe curious works of antiquity; and it appears, that ſoon after he was placed in the Mathematical Chair, he ſet about the inveſtigation. From his papers ſtill remaining, we learn that his endeavours were firſt directed to the improvement of the defective reſtorations of these books, by preceding geometers. From the ſame ſource alſo we know, that within a few years he turned his attention to the Poriſms of EUCLID; and it was to be expected that the curious nature of theſe Propoſitions, and even the difficulty of the inveſtigation, would attract the early notice of his ardent mind.

Unfortunately the deſcription of the Poriſms by PAPPUS, in all the MSS. of his *Collections* which have been examined, is ſo mutilated, that every attempt to reſtore them, before Dr. SIMSON's time, had failed. The firſt part of the deſcription, which ſeems to be entire, is calculated only to excite curioſity; being too general for conveying any preciſe notion of theſe Propoſitions, or for giving any effectual aſſiſtance for the recovery of them: and the remainder, containing a detail of the contents of EUCLID's work, is through the whole ſo depraved by the injuries of time, that all endeavours to explain it were nugatory. Some Geometers indeed, of great name, flattered themſelves that they had got poſſeſſion of the ſecret of this peculiar claſs of Propoſitions; but ſubſequent to them, Dr. HALLEY, with all his genius, his extenſive knowledge, and his ſucceſsful experience in unravelling ſome other pieces of ancient geometry, gives up the Poriſms as a hopeleſs purſuit; and he admits that the deſcription of PAPPUS, as it now ſtands, is unintelligible and uſeleſs.*

Dr. SIMSON, however, fully proved that thoſe ingenious men before Dr. HALLEY, who ſuppoſed that they had diſcovered the Poriſms, had certainly deceived themſelves; and that the celebrated FERMAT alone, in modern times, had acquired ſome notion of the nature of Poriſms, but without being able to unfold it compleatly; and without having reſtored any one Propoſition, that could be ſuppoſed to belong to the Treatiſe of EUCLID.

The Doctor himſelf candidly informs us, how long and how ſeriouſly he had ineffectually laboured in ſearch of this ænigma, as the nature of a Poriſm truly became, from the very mutilated ſtate of the only exiſting deſcription of it. An ardent curioſity was a prominent and well-known part of his character; and in this caſe, his curioſity was enlivened by his predilection for the ſtudy of the ancient geometry; but we may preſume alſo, that a natural and laudable ambition for the diſtinction which would reſult from ſucceſs, in a purſuit in which the greateſt Mathematicians of his own and of the preceding age had failed, would animate his zeal and perſeverance in the inveſtigation. He had been occupied on this ſubject even in the year 1715, and perhaps before that period; for he obſerves, that in that year he had demonſtrated the firſt caſe of FERMAT's fourth Poriſm, before he had acquired the knowledge of the nature of that claſs of Propoſitions.*

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* "Hactenus Poriſmatum Deſcriptio, nec mihi intellecta nec lectori profutura." See the Preface to the Seventh Book of PAPPUS, edited by Dr. HALLEY, octavo, 1706, page 87, note at the end of the Poriſms.

The firſt direction of his reſearch ſeems to have been, to endeavour to diſcover the Poriſms, from the general deſcription of them in the beginning of the account given by PAPPUS; and when this failed, he tried to recover ſome of the individual Poriſms, from which he hoped to aſcertain the diſtinctive qualities of theſe Propoſitions; but in this attempt he had no better ſucceſs. He continued his reſearches, and devoted his whole attention to the ſubject. For a conſiderable time his imagination was compleatly occupied by it: his mind was harraſſed by the conſtant, but unſucceſsful exertion; he loſt his ſleep, and his heath was injured: but all his endeavours were ineffectual; and therefore he finally determined to baniſh for ever the ſubject from his thoughts.

For ſome time he maintained this reſolution, and applied himſelf to other purſuits;† but afterwards he happened to be walking with ſome friends on the banks of the river Clyde at

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* SIMSON's Poſthumous Works, p. 540, laſt line.

† It is not improbable, that Dr. SIMSON about this time turned his attention to Algebra, and that, on deſpairing of the Poriſms, he had employed himſelf in the inveſtigation of Serieſes for the Circle, of which an account is given in this Memoir, (ſee Note H.) The Poriſms, as I reckon, were firſt diſcovered in April 1722. For ſome time after, it is reaſonable to ſuppoſe that he was almoſt entirely occupied in proſecuting his invention; and therefore, as he tranſmitted theſe Serieſes to Dr. JURIN in February 1723, and from his letter alſo it appears that they had been found out ſome time before, it is very probable they were inveſtigated before his invention of the Poriſms. They may be conſidered as a reſpectable ſpecimen of what Dr. SIMSON might have done in Algebra, had he devoted his attention to that branch of Mathematics.

Glaſgow, and by accident being left behind his company, he inadvertently fell into a reverie reſpecting the Poriſms. Some new ideas ſtruck his mind, and with his chalk having drawn ſome lines on an adjoining tree, at that moment, for the firſt time, he acquired a juſt notion of one of EUCLID's Poriſms.* I have repeatedly heard him relate the occurrence, which he ſeemed to do with pleaſure, and he mentioned even the ſcite of the tree on which he deſcribed the fortunate diagram.†

After this firſt diſcovery, however, it required time and much inveſtigation, before he could reſtore, to his ſatisfaction, the general Propoſition of EUCLID's firſt book of Poriſms; and it appears that his firſt communication of the diſcovery to the Mathematicians of London was through Mr. MACLAURIN. Juſt before Mr. MACLAURIN's ſetting out from Scotland for France, by the way of London, in 1723, Dr. SIMSON communicated to him the Propoſition he had recently reſtored, and which a ſhort time after was printed in the Philoſophical Tranſactions. It appears alſo that Dr. JURIN, then Secretary of the Royal Society, had mentioned in a letter to Dr. SIMSON this communication from Mr. MACLAURIN, for Dr. SIMSON, in a letter to Dr. JURIN, dated Feb. 1, 1723, in reply to one he had juſt received from him, ſays, "The Propoſition of "PAPPUS, which I ſhewed Mr. MACLAURIN when here,* is "the general Propoſition into which PAPPUS collected the "Poriſms of the firſt book of EUCLID's Three; which, together "with the more general Propoſition immediately ſubjoined, "and which are both deficient, (I mean imperfect,) I have, "with no little inveſtigation, recovered and demonſtrated, "together with ſome few of the Poriſms; the reſt I have not had "leiſure enough to try. I mean thoſe of the firſt book, for as "to thoſe of the two others, excepting what may be included "in the ſecond of the above-mentioned Propoſitions, I believe "it will be extremely difficult for any body to reſtore them. If "I underſtand you like to ſee them, I ſhall ſend them as ſoon "as I can get the ſcribbles I have about them wrote over fair."†

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* This incident is generally ſtated by Dr. SIMSON himſelf, in his Preface to the Poriſms. Poſt. Works, pp. 319, 320.

† Though moſt of Dr. SIMSON's notes and propoſitions are dated, yet there is no entry to be found among his papers of this particular incident. From the dates of ſeveral of his firſt inveſtigations of the Poriſms, both in his PAPPUS, and in other MSS. it had, with much probability, occurred in April 1722; for in that month are ſome of his firſt notices about the Poriſms. At the end of one of them he adds, "Hodie hæc "de Poriſmatis inveni, R. S. April 25, 1722." Alſo in a note on Prop. 131, lib vii. PAPPI, he ſays, "Poſtquam vero ipſa Poriſmata, ea velim quæ generali propoſitione "in præfatione ad hunc librum ſeptimum, complexus eſt PAPPUS, multo labore at ſuc-"ceſſu præter quem ſperare æquum fuit felici, tandem ex manca et contracta "admodum PAPPI deſcriptione inveſtigavimus; facilius erit lemmata hiſce inſer-"vientia, ad priſtinum nitorem reſtituere. April 27, 1722. ROB. SIMSON."

It would appear that the reſult of this letter was an immediate communication of the Propoſitions to Dr. JURIN, which, as might be expected, were much approved by thoſe eminent Geometers Dr. HALLEY, Mr. MACHIN, and Dr. JURIN himſelf. In the courſe of the ſame year the Paper was printed in the Philoſophical Tranſactions,‡ and Dr. SIMSON, in a letter to Dr. Jurin, of January 10, 1724, expreſſes his ſatisfaction at the reception of his communication in the following manner: " The honour you have done me in printing the Paper I ſent " up, in the Philoſophical Tranſactions, is what I am very " ſenſible of; and you may be ſure the approbation any thing " in it has had from ſuch good judges as Dr. Halley, Mr. " Machin, yourſelf, and the other learned gentlemen you " were pleaſed to ſhew it to, cannot but give me a great deal " of pleaſure; and nothing could excite me more to endeavour " to reſtore the other Poriſms, of which there are the leaſt " data: but I find it a very difficult affair, eſpecially to " one who is ſo ſlow, as by much experience I find myſelf. " However I ſhall at leiſure hours, God willing, try what I " can do. I deſire you may give my humble reſpects to Dr. " Halley and Mr. Machin; and be pleaſed to tell the Doctor, " that as to the meaning of the words Ἐὰν ὕϖτιȣ ἢ ϖαρυϖτίȣ, " ἢ ϖαραλλήλȣ, I never was ſolicitous about the meaning of " the propoſition, but at his deſire I have conſidered the paſſage," &c. Dr. Simson then enters into a long and learned diſcuſſion of the proper meaning of this paſſage, of which the reſult is ſhortly ſtated in a note, page 348, Poſt. Works.*

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* This is mentioned in the Preface to Dr. SIMSON's Conics, p. vi. 2d edit. See alſo Note A. at the end.

† The principal object of this letter was to communicate to Dr. JURIN ſome Serieſes about the Circle, of which notice will be taken afterwards. See alſo Note H. at the end.

‡ Vol. XL. for 1723, p. 330. In the ſame volume of the Tranſactions, p. 248, is a Paper by Dr. PEMBERTON, containing ſome propoſitions about the Rainbow; at the

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beginning of which he obſerves, " For the greater brevity, I ſhall deliver them under " the form of Poriſms; as, in my opinion, the ancients called all Propoſitions treated " by analyſis only." This intimation naturally gave ſome diſſatiſfaction to Dr. Simson, as aſſuming the appearance of being the firſt to announce to the world the nature of the ancient Poriſms. In the copy of this volume of the Tranſactions belonging to the College Library of Glaſgow, the Doctor wrote a ſhort animadverſion on the margin of the page, which concludes with theſe words: " but he (Dr. Pemberton) has " entirely miſtaken the nature of a Poriſm, and his two Propoſitions have neither the " form nor matter of Poriſms. R. S."

* The reſt of the letter may be intereſting to ſome readers, and is therefore placed in Note C, at the end.

For a few years after his diſcovery of the Poriſms, his mind was much engaged in the further proſecution and illuſtration of it. It appears, however, from his papers, that at this very time he occaſionally applied to other branches both of ancient and modern Mathematics; though, as was to be expected, the chief object of his ſtudy was the improvement and extenſion of his inveſtigation of the Poriſms, by recovering more of Euclid's Propoſitions, and adding alſo ſome of his own, and ſubſequently alſo the contributions of a few mathematical friends, to whom he communicated the intereſting intelligence of his diſcovery. There are many indications of his intention of publiſhing the Poriſms; but from various cauſes he poſtponed the execution of it, till, in the progreſs of life, he had acquired ſo very ſtrong an impreſſion of the decline of his faculties, that he reluctantly gave up the deſign. The *Treatiſe on Poriſms*, however, with ſome other valuable tracts, were publiſhed a few years after the Doctor's death, at the ſole expenſe of his highly reſpected and learned friend the late Earl Stanhope; and by the full explanation of the Poriſms contained in that volume, (of which I ſhall have occaſion to give a more particular account,) this hitherto inexplicable portion of ancient geometry was, by the Doctor's perſeverance and ingenuity, compleatly reſtored and explained. It may juſtly be admitted, that the ſatisfaction and the pride of invention, in the elucidation of this very difficult branch of ancient ſcience, involved in ſuch obſcurity from the depraved ſtate of the only remaining account of it, were properly high; and by liberal and ſcientific minds, the warmth with which he expreſſes theſe feelings will not be diſapproved: "Deſcriptio "autem quam tradit (Pappus) Poriſmatum adeo brevis eſt et

“ obſcura, et injuria temporis aut aliter vitiata, ut niſi DEUS “ benigne animum et vires dederat in ea pertinaciter inquirere, “ in perpetuum forſan geometras latuiſſent.”[*]

It has been already obſerved, that before Dr. SIMSON obtained any juſt notion of the Poriſms, he had begun to improve the reſtorations of other ancient geometrical treatiſes; which had been attempted by preceding geometers, particularly of the *Loci Plani*, and *Sectio Determinata*,[†] of APOLLONIUS. To thoſe who have ſome knowledge of this ſubject, it is needleſs to explain the value of theſe two treatiſes; particularly of the former, as containing many elegant Theorems, and as being eminently uſeful in the reſolution of Problems. The *Loci Plani* of APOLLONIUS had been reſtored in a certain manner, both by FERMAT, and by FRANCIS SCHOOTEN. In the poſthumous works of the former very diſtinguiſhed Mathematician,[‡] is given a reſtoration, geometrical indeed, but ſynthetical only without analyſis, and deficient alſo in other material points, particularly in the diſtinction of the caſes, and in aſcertaining the determinations; without which no geometrical reſolution can be conſidered as compleat.[§]

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[*] Opera Reliqua, p. 513.

[†] The *Sectio Determinata* was not publiſhed in his own time, and I therefore defer any notice of it, till an account be given of the volume of Poſthumous Works.

[‡] FERMAT had reſtored this work of APOLLONIUS before 1629, as appears by a letter of his, p. 153 of his Works. But it was printed only in 1679, among his Poſthumous Works. See FERMAT. Oper. Varia, tom. 11. p. 12. FERMAT had not ſeen the Prefacė to the 7th book of PAPPUS in Greek.

[§] The uſe of *Loci* in the reſolution of Problems is very obvious, even to thoſe who are but little acquainted with ancient Geometry. The great importance of the compleat diſtinction of caſes and determinations in thoſe treatiſes of the τόϖος ἀναλυ-

The reſtoration by SCHOOTEN has ſimilar defects; in a few only of the problems an analyſis is given, but one purely algebraical; and he acknowledges in his preface* that his reſtoration of the *Loci Plani* was deſigned to be an illuſtration of the geometry of DES CARTES, by furniſhing proper examples of his method. Though it appears that Dr. SIMSON had begun, at a very early period, to reſtore the *Loci Plani* after the ancient model, and though the work was almoſt compleated before he publiſhed his *Conic Sections* in 1735, yet it was not printed till 1749. What reaſons occurred for this delay cannot now be concluſively aſcertained; but we learn from his papers, and from ſome remains of his correſpondence, that at one time he had deſigned to add one or two books of *Loci* to thoſe of APOLLONIUS. There are many detached Propoſitions, and even ſome Serieſes of Propoſitions of that deſcription, in which the two books of APOLLONIUS are quoted as part of the ſame work, which ſufficiently aſcertains his purpoſe, at the time of writing them.† It is indeed much to be regretted, that he did not purſue this

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ομένος, which conſiſt of general Problems, to which other Problems may often be reduced, will be explained afterwards. The ſame attention to the caſes and determination of *Loci*, and alſo of Poriſms, is equally neceſſary, to render the application of them uſeful to the ſolution of Problems.

* F. SCHOOTENII *Exercitationes Mathematicæ*, Præf. 1657.

† In a letter to Earl STANHOPE, dated September 10, 1750, after mentioning that the *Loci Plani* were juſt printed, and about to be publiſhed, he adds, "that he once "deſigned to have added ſeveral other *Loci Plani*, but thought it beſt now to give "only thoſe mentioned by PAPPUS, with a few to make ſome of them more com"pleat."——There is alſo among his papers a ſketch of a title for the *Loci Plani*, comprehending ſome additions to thoſe of APOLLONIUS.

ſcheme; as no work could be more generally uſeful in geometrical inveſtigations, than an extenſive and well-arranged collection of *Loci*. The Doctor's plan ſeems alſo to have extended to ſolid *Loci*, of which indeed he gives a hint, in the preface to his *Conic Sections*; and among his papers are alſo ſome ſmall Sets of ſuch Propoſitions, which indicate the probability of the deſign which I ſuppoſe him to have entertained.* The heſitation about making additions to the Treatiſe of APOLLONIUS probably contributed to the delay in printing the work, which was not executed till 1749. He then met with ſome unexpected difficulties in treating with a bookſeller for the ſale of the whole impreſſion, which alone prevented the publication at that time;† and except a few copies diſtributed among his friends in 1750, the book remained unpubliſhed, till after his death. Such is the elegance of method, and the ingenious contrivance of demonſtration in this work, that he has truly exhibited a copy, or at leaſt ſo very nearly a copy, of the work of APOLLONIUS, that little regret need be had for the loſs of the original. The preface alſo is well deſerving the attention of thoſe who wiſh to acquire juſt notions of the ancient books of analyſis.

The firſt publication by Dr. SIMSON, except the Paper on Poriſms in the Philoſophical Tranſactions, was his Treatiſe *Sectionum Conicarum, libri v.* which appeared in 1735.

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* In a great many of the Propoſitions of ſolid *Loci* the correſponding algebraical equations are added. From ſome obſervations in the preface to the ſeventh book of PAPPUS, it appears that the ancients had conſidered ſeveral varieties of *Loci*, particularly the plane, the ſolid, the linear, and thoſe called *Loci ad medietates*, ariſing from mean proportionals; but of theſe laſt only a very few imperfect notices remain in the beginning of the third book.

† This account is given by Dr. SIMSON, in a letter to the late Earl STANHOPE, Sept 10th, 1750.

He had obſerved, in the firſt years of his ſtudy of Mathematics, that the Treatiſes on Conic Sections, then in moſt general uſe and eſtimation, were entirely algebraical; and the great merit of the work, written in that ſtile by the Marquis DE L'HOSPITAL, contributed not a little to the popularity of this mode of treating geometrical ſubjects. It occurred therefore to Dr. SIMSON, that a treatiſe on Conic Sections, written on the purer model of antiquity, might have ſome influence in correcting the prevailing falſe taſte, of introducing algebraical calculation into thoſe branches of geometry where it was not neceſſary, and where it ſupplanted a more elegant form of analyſis and demonſtration. To exhibit, therefore, a juſt compariſon of the two methods, he aſſumed the ſame definitions of the Conic Sections, as L'HOSPITAL and others before him had employed; and from them, with the true ſimplicity and accuracy of the ancient ſchool, he deduced not only the properties of theſe curves as given by all preceding writers, but added many new and important Propoſitions of his own,* with the generalization and improvement of many, which had been previouſly diſcovered. It is unneceſſary to enter into any particular diſcuſſion of the merits of a work, which has been for ſo many years before the public, which was received with general approbation on its appearance, and which, notwithſtanding more recent improvements, ſtill maintains the reputation, of being the beſt example of the ancient method of demonſtration, of having communicated large additions to the theory of the Conic Sections as it ſtood in his time, and of exhibiting a clear and juſt arrangement of the properties of theſe curves.

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* The preciſe object which Dr. SIMSON had in view, was the occaſion of his adopting the definitions of the curves given by L'HOSPITAL and others, in which the language is rather mechanical, than of a ſtrictly ſcientific character. This I have frequently heard the Doctor expreſs; and he obſerved at the ſame time, that he conſidered the derivation of the properties of theſe curves from the cone, after the original method of the ancients, as the beſt; and that if a definition of them, from a deſcription in a plane, were to be aſſumed as moſt expedient, a more correct form of language than L'HOSPITAL's ought to be uſed; which indeed is generally done by thoſe modern geometers who have relinquiſhed the conſideration of the cone, in defining this claſs of curves.

This Treatiſe became a part of Dr. SIMSON's Courſe of Lectures in the College; it was reprinted in 1750 with ſeveral additions, including ſome valuable communications by Dr. MATTHEW STEWART; and the three firſt books have been tranſlated, and repeatedly printed, as an elementary introduction to this branch of ſcience. In the preface, Dr. SIMSON gives a ſhort but ſatisfactory ſketch of the hiſtory of this portion of geometrical ſcience from the age of MENÆCHMUS, reputed the firſt inventor, to his own time; in which ſome omiſſions have been remarked, but not of much importance in ſo ſhort an abſtract, as it was his purpoſe to communicate.*

To the ſecond edition of the *Conics* is added an appendix, containing two Geometrical Problems, with a preliminary *Locus*; which Dr. SIMSON gives as examples of the ſuperiority of the ancient analyſis over that of the moderns, in the reſolution of the ſame problems, by the authors there quoted. That ſuperiority will be generally admitted; but it is proper to remark, that beſides the ſolution of one of the problems by GUISNEE,[†] referred to by Dr. SIMSON, this author, in the ſame Treatiſe, gives another very elegant conſtruction of the ſame problem, of triſecting the arch of a circle, by means of two local equations.[*] The Doctor's object, however, was to ſtate a compariſon between the pure geometrical analyſis, and the uſual algebraical method, of reſolving an equation, and conſtructing that ſolution. The other method by the combination of *Loci*, though clothed in an algebraical dreſs, Dr. SIMSON would have conſidered, and with reaſon, as in effect geometrical; for the *Loci* in queſtion might have been eaſily deduced geometrically, and thence the proper geometrical ſolution became obvious.[‡]

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* See Note A. at the end.

In the year 1752, Dr. SIMSON tranſmitted to the late Earl STANHOPE, in return for ſome valuable communications from his Lordſhip, an inveſtigation of a rule of approximation to the roots of numbers which are not perfect ſquares, given by ALBERT GIRARD in 1629, without demonſtration, in his edition of the works of STEVINUS. That rule ſeems to have eſcaped the notice of Mathematicians, till Dr. SIMSON undertook the inveſtigation of it; which, on the recommendation of Earl STANHOPE, was ſubmitted to the Royal Society, and publiſhed in the Tranſactions of that year. The method is ingenious, and in ſome caſes may be uſeful; though, from modern analytical improvements, ſuch rules become of inferior conſequence.

[†] GUISNEE *Appl. de Algebre à la Geometrice*, p. 191, 2d ed.

[*] The ſame Problem is reſolved with great ſimplicity by BOSCOVICH, by means of an Hyperbola, and the circle to which the given arch belongs: ſee MAKO *de Arithm. et Geomet. Æquationum Reſolutionibus*, p. 332. It may not be improper to mention here a remark by Dr. SIMSON, in a letter to the late Earl STANHOPE, reſpecting a Problem propoſed by the Doctor's ingenious pupil Mr. WILLIAMSON of Liſbon. "As to the "ſubſtituting a circle in place of one of the Hyperbolas, I never tried to do it in this "Problem, becauſe I obſerved the ancients were not ſolicitous about ſuch ſolutions, "and preferred the *Loci* which naturally ariſe from a Problem to any other, as afford-"ing the ſhorteſt compoſition. Thus in the 58th Prop. of the 5th Book of APOLL. "*Conica*, though it is not difficult to ſolve the Problem by a circle and the given "Parabola, yet APOLLONIUS takes the Hyperbola that ariſes from the Problem as "giving the moſt natural and ſhorteſt ſolution." The letter is dated March 7, 1755.

[‡] Dr. SIMSON's opinions on this ſubject are more fully ſtated in his correſpondence with GEORGE LEWIS SCOTT, Eſq; for which ſee Note K. at the end.

The next and only other work of Dr. SIMSON which was publiſhed in his life time, was his excellent edition of EUCLID, which appeared both in Latin and Engliſh in 1756, and was dedicated to his preſent MAJESTY when Prince of Wales. It contained the firſt ſix books of the *Elements*, with the eleventh and twelfth, being thoſe uſually taught in Univerſities. From his annually lecturing on theſe books, and his accurate knowledge of the ancient geometry, many corrections of the common text of EUCLID would naturally occur to him; but it was only after many and repeated ſolicitations from his mathematical friends, that he was induced to undertake the preparation of a new edition. Even after he had nearly compleated it, a conſiderable delay occurred, from doubts which were then entertained reſpecting the conſtruction and effect of the Statute of Queen ANNE, for the encouragement of learning. He was flattered by his friends with the hope that an edition corrected by him would have an extenſive circulation, and would nearly ſuperſede the common editions then in uſe; and he was therefore naturally anxious to ſecure the advantages of his work, at leaſt for the term preſcribed by the Statute. But from ſome of his letters to the Earl STANHOPE, it would appear that ſuch were the apprehenſions entertained on this ſubject, particularly about the edition in Latin, that he had it in contemplation to ſolicit a ſecurity of his right by a private Act of Parliament; and the Act in favour of Mr. BUCKLEY, the Editor of *Thuanus*, was quoted as a precedent.* The Speaker ONSLOW was conſulted; but on full conſideration, this ſcheme was relinquiſhed, and Dr. SIMSON relied on the protection of the Statute.

To judge with impartiality of the merits of this work, the ſtate of the text in preceding editions muſt be attended to. Dr. SIMSON, from his veneration for the ancient Geometers, ſeems, with an excuſable partiality, to have aſſumed, that the *Elements* of EUCLID, as they came from the author, were nearly without blemiſh; and he therefore aſcribes all the errors and imperfections of the common editions, either to the careleſſneſs of tranſcribers, or to the blunders of THEON, and other ancient Editors. His corrections are numerous, and many of them important; and even now, when moſt of them are adopted, it might be an uſeful exerciſe for the young mathematician to ſtudy the grounds of his emendations, which exhibit ſo clearly the preciſion of his ideas, and the logical accuracy of his underſtanding. Some animadverſions were made on this edition, chiefly by thoſe whoſe works had been criticiſed in the Doctor's notes; and to ſome of theſe, in a ſecond edition, replies and explanations were made; but he had a great averſion to controverſy, and his obſervations on what he had proved to be errors or defects in his predeceſſors, were never calculated to provoke it.†

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* This Act was for preventing for fourteen years the importation of any Latin copies of that work; which was not provided for in the Statute of Queen Anne.

Notwithſtanding Dr. Simson's valuable corrections, there are ſtill ſome difficulties in the *Elements*, which remain to be cleared up by ſome future Editor. The demonſtration of the property of parallel lines (29 I. Elem.) is ſtill theoretically defective, requiring the admiſſion of ſome principle, not ſtrictly belonging to the claſs of ſelf-evident truths. It has by ſome been ſuppoſed, that the remedy for this difficulty muſt be ſought for in a juſt definition of a ſtraight line. No definition of a ſtraight line has yet been found, and none perhaps can be found, from which all the properties aſſumed in the *Elements* to belong to it, can be rigidly demonſtrated. There is manifeſtly alſo ſome defect in the definition of a ſolid angle, ſince what is given in Dr. Simson's, and in all other editions, does not diſcriminate the ſolid angle from a number of plane angles, formed at one point, which may exiſt according to the definition, but without forming the ſolid angle intended to be defined. The improvements and corrections of the Fifth Book are alſo important. His obſervation with reſpect to ſolid figures, in the note on Def. 10. XI. Elem. is curious, from remarking an error, which is ſo obvious when pointed out, but which had eſcaped the notice of the many learned and acute Geometers, who had paid much attention to EUCLID's *Elements*. An obſervation of a ſimilar kind, and about the ſame time, was made by Mr. LE SAGE, which is recorded in the Hiſtory of the Royal Academy of Sciences at Paris for 1756; and another important correction has been more recently made by LE GENDRE, of which a ſatisfactory hiſtory is given by Mr. PLAYFAIR, in the ſecond edition of his *Elements of Geometry*.

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† Soon after the publication of Euclid, Sir Andrew Mitchel, then the Britiſh Miniſter at the Court of Berlin, aſked Dr. Simson's permiſſion to preſent, in the Doctor's name, a copy of the Latin Euclid to Frederic III. The Doctor was gratified by the requeſt, and tranſmitted to Sir Andrew Mitchel a copy of the book, on the page oppoſite to the title of which was written the following inſcription:

"Cesari in belligerando, Ptolemæis in promovendis artibus et ſcientiis, animumque ſuum doctrina excolendo merito æquiparandus, Magnus Imperator FREDERICUS Tertius, Borussorum Rex, ut librum hunc benigne accipiat ſumma veneratione exoptat. Robertus Simson, Editor."

The Book of EUCLID's *Data* was annexed by Dr. SIMSON to a ſecond edition of the *Elements* in octavo, (1762,) with many neceſſary corrections, and ſome valuable additional Propoſitions, both of his own and of his learned friends the late Earl STANHOPE, and Dr. STEWART. This book is one of the thirty-three, compoſed by the ancient Geometers, for facilitating the reſolution of problems by their analyſis; and is the moſt proper introduction to the ſtudy and practice of that analyſis.* At the end, Dr. SIMSON has added two geometrical Problems, for illuſtrating the uſe of ſome Propoſitions of the *Data*, which would not be obvious to beginners; and with reſpect to one of them he obſerves, that he believes "it would "be in vain to try to deduce the preceding conſtruction from "an algebraical ſolution of the Problem." The obſervation was perhaps haſtily made, when he was ſeventy-five years of age; but he plainly had in view the moſt common method of reſolving geometrical Problems by algebra, viz. by deducing a final equation with only one unknown quantity, reſolving that equation, and conſtructing the ſolution of it; and with reſpect to this method, the remark would be juſt. An ingenious friend of the Doctor's, the late GEORGE LEWIS SCOTT, eſq; intimated to him that a conſtruction, ſimilar to his, might be derived from the combination of two *Loci*, expreſſed by two equations ariſing in the algebraical ſolution of the problem. But this communication, with the Doctor's reply, is inſerted in a note at the end, which will be more ſatisfactory than any abſtract of it in this place.*

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* In a letter of Dr. SIMSON's, in Note K, are ſome obſervations on the uſe of this book.

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* See Note K.

SECTION III.

Of Dr. SIMSON's *Poſthumous Works.*

THE ſtrong impreſſion which Dr. SIMSON felt of the failure of his memory, having prevented his publiſhing ſome important Geometrical Works; the copies of theſe works, with a large maſs of miſcellaneous papers, fell at his death, into the hands of his friend and executor Mr. CLOW, the Profeſſor of Logic in the College of Glaſgow.†

While Mr. CLOW was deliberating what was moſt expedient to be done with regard to theſe papers entruſted to his care, the late Earl STANHOPE, diſtinguiſhed in his elevated rank by his ingenious cultivation and liberal patronage of the Mathematical Sciences, intimated his deſign of publiſhing thoſe works of Dr. SIMSON which he had compleated, with any other pieces, which though unfiniſhed, might without injury to his fame, be given to the public. The munificent propoſal was moſt acceptable to Mr. CLOW; and after ſome correſpondence reſpecting the ſelection, a large volume, in the year 1776, was, at his Lordſhip's ſole expenſe, handſomely printed, under the care of Mr. CLOW, and liberally diſtributed.

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† A conſiderable portion of theſe papers conſiſts of various firſt ſketches of his works, publiſhed by himſelf, or ſince his death.

This volume contains a reſtoration of the *Sectio Determinata* of APOLLONIUS, with two additional books by Dr. SIMSON; and a full explanation of the Poriſms, with a reſtoration of a number of the Propoſitions of EUCLID's original work. To theſe two important works are added ſome ſmaller Tracts, which ſhall be taken notice of in the order in which they are placed in the volume.

The two books of *Sectio Determinata*, it was formerly obſerved, had engaged Dr. SIMSON's attention at an early period of his life. That treatiſe indeed, had been reſtored by SNELLIUS, but in a very imperfect manner, without the neceſſary diſtinction of the various ſituations of the Points, called (by APOLLONIUS) *Epitagmata*; and without a compleat expoſition of the Determinations, which, as is well known, are neceſſary to the perfect ſolution of any Problem; and are more particularly requiſite, as has already been obſerved, in the books forming the τόϖος ἀναλυομένος, to render them uſeful for the purpoſe for which they were compoſed and collected.*

In a long and inſtructive preface Dr. SIMSON explains the many defects in the Work of SNELLIUS, which need not be here enumerated.† He takes notice alſo of the ſubſequent reſolutions of ſome of the Problems, by ALEXANDER ANDERSONUS;‡ and of thoſe likewiſe contained in the Treatiſe of Geometrical Analyſis by Hugo de Omerique; and in the work itſelf he adopts ſome Propoſitions from theſe performances. In the Doctor's firſt attempts to reſtore the *Sectio Determinata*, the difficulty was not ſo much the giving proper and even elegant ſolutions of the various caſes of the general Problem, as the finding the accurate diſtinction of the *Epitagmata*, the aſcertaining the determinations; and chiefly the inveſtigation of ſolutions, which required the uſe of the *Lemmata* aſſumed by Apollonius, and demonſtrated by Pappus,* by which the identity of the reſtoration with the original work might be recognized. In the preface he obſerves, that it was not till 1727,† and after many unſucceſsful attempts, that he obtained ſuch ſolutions of the fifth and ſixth Problems of the firſt book. The remaining part of the work required much inveſtigation, and amidſt the various other ſtudies in which he was engaged, it was not till above twenty years after, that he compleated the reſtoration to his entire ſatisfaction, and in ſuch a manner as to leave no doubt of its being truly the ſame as the original of Apollonius; or ſo nearly the ſame, as to preclude the occaſion of any further enquiry reſpecting it.‡

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* See the Preface to Dr. SIMSON's *Reſtoration of the Loci Plani*, pp. 7 and 8; where the uſe of the books of the τόϖος ἀναλυομένος in reſolving Problems is briefly but very clearly ſtated.

† It is amuſing to obſerve SNELLIUS blaming PAPPUS for placing the *Sectio Determinata* after the *Sectio Rationis*, in conſequence of his own defective method of reſtoring the latter treatiſe by means of the former.

‡ In *Supplemento* APOLLONII *Redivivi.* Paris, 1612.

To the reſtoration of the work of Apollonius, two other books, containing an extenſion of the general Problem, are added by Dr. SIMSON, of which it is neceſſary for me ſhortly to ſtate the hiſtory. About two years before his death, when converſing with him on ſome geometrical ſubject, he mentioned that he had compleated the reſtoration of the *Sectio Determinata.* He informed me alſo, that he had written two additional books, a fair copy of which he took out of his repoſitory, and gave me in confidence, with an injunction to publiſh them, or not, according to the reception which his reſtoration of the work of APOLLONIUS might meet with among mathematicians. At that time he was probably thinking of publiſhing the work of APOLLONIUS, but he did not enter into any particular explanation of his intentions.

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* For a more particular account of theſe *Lemmata*, ſee the Appendix.

† *Opera Reliqua*, Præfat. ad *Sec. Determ.* p. v.

‡ In a letter to Earl Stanhope, Sept. 10, 1750, he mentions, that for more than twenty years he had been endeavouring to find the uſe and application of the *Lemmata* in Pappus, in the ſolution of the caſes of the *Sectio Determinata*, but had not done ſo completely till laſt ſummer. And among his papers there is an intimation of his having juſt completed the reſtoration of the *Sectio Determinata*, by the uſe of the *Lemmata* of Pappus: to which he adds, "a Geometris intacta ſunt a tempore Pappi "in hunc uſque diem. 28 Jan. 1749. R. S."

During his laſt illneſs, and for ſome time before it, I happened to be at a great diſtance from him, and he died without my receiving any further communication on the ſubject. A ſhort time after his death, therefore, I conſidered it to be my duty, after explaining theſe circumſtances, to deliver the manuſcript to Mr. CLOW, his particular friend, to whom he had aſſigned by his will the property and charge of all his papers. An account of this ſupplement being afterwards communicated to the late Earl STANHOPE, his Lordſhip was pleaſed to deſire that it ſhould be annexed to the Doctor's reſtoration of the work of APOLLONIUS.* From the nature of the ſubject, and the conſtant references to the two former books, the additional books are rather unintereſting, eſpecially when read without any particular application of them to the ſolution of Problems. But they have a ſimilar utility to that of the former books; and when any geometrical Problems can be reduced to ſome particular caſes of this ſupplement, it affords alſo an immediate conſtruction and demonſtration of ſuch Problems.*

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* At the end of a former copy of the reſtoration of this work of APOLLONIUS is the following. N. B. "Unicuique Problematum quæ in duobus de *Sectione* "*Determinata* Libris continentur, addenda ſunt tanquam corolloraria in proprijs "locis, Problemata in quibus quadrata vel rectangula, ſunt quadratis vel rectangulis "majora vel minora, dato quam in ratione; vel quorum unum ſimul cum eo quod ad "alterum datam habet rationum, datum eſt. Nam et hæc ſæpius in uſu veniunt." But inſtead of this, the Doctor had afterwards thought proper to expand the Problem in the two additional books juſt mentioned.

The next, and certainly the moſt important portion of the poſthumous volume, contains Dr. Simson's diſcovery and illuſtration of the Poriſms of Euclid; and as this volume is not in general circulation, it may be a gratification to ſome readers, beſides the account already given, to add a more particular detail of the hiſtory of this curious piece of ancient ſcience, and of the Doctor's ſucceſsful inveſtigation of it, after it had baffled the reſearches of all the ingenious men who attempted it before him.

The ſeventh book of the *Mathematical Collections*, it has been obſerved, is the only notice to be found in ancient authors of Euclid's Poriſms; if we except the few ſhort and not very ſatisfactory obſervations of Proclus, who lived, as is ſuppoſed, within a century after Pappus.† The tranſlation by COMMANDINE of the ſix laſt books of PAPPUS, (which were all that were found in the MS. uſed by him) was publiſhed in 1588.—The attention of the Mathematicians of Europe was ſoon directed to that work; and it gave riſe to the various attempts which were made to reſtore the loſt works of the ancient Geometers, ſo particularly deſcribed in the preface to his ſeventh book. The Poriſms of courſe were not neglected; though the very imperfect ſtate of the deſcription of them by PAPPUS, in all the manuſcripts which have hitherto been examined, created peculiar, and apparently unconquerable, difficulties in the inveſtigation.

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* Since Dr. Simson's death two other reſtorations of the *Sectio Determinata* have been publiſhed. One by W. Wales, in 1772; and another, by Giannini, for which ſee *Opuſcula Mathematica, auctore* Petro Giannini, Parmœ, 1773.—But with reſpect to them, it is ſufficient to obſerve, that independently of the curious information in Dr. Simson's valuable preface, the ſuperiority of his work, as a reſtoration of Apollonius, muſt be obvious to the geometrical reader.

† Pappus flouriſhed as is generally reckoned about the year 400 of the Chriſtian Æra, and Proclus towards the year 500, and the former is repeatedly quoted by the latter. See Suidas, and Gerard. Vossius *de Univerſæ Matheſeos natura et Conſtitutione, et Chronologia Mathematicorum.*

ALBERT GIRARD, a Geometer of eminence in the early part of the 17th century, was the firſt who announced the reſtoration of the Poriſms of EUCLID. In his *Trigonometry* (1629), and alſo in his edition of *Stevinus* (1634,) he ſtates his having reſtored the Poriſms; but in terms ſo general, that no preciſe opinion can thence be formed of his notions on the ſubject. But from the firſt of theſe intimations Dr. SIMSON reaſonably infers, that GIRARD was not acquainted with the true nature of the ancient Poriſms; and it appears alſo to be highly improbable, if he had had any ſucceſs in recovering ſo curious a branch of ancient learning, that he would have concealed it from the world.*

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* Dr. SIMSON in his preface juſtly obſerves, that the ſhort notices of Poriſms in C. RENALDINUS's work *De Resol. et Comp. Math.* (Bon. 1644) are of no uſe in explaining the ancient Poriſms, and in truth have no relation to them. He ſeems to uſe the term Poriſm for expreſſing the general rule reſulting from an algebraical inveſtigation in general terms: ſee pp. 280, 1. They are general corollaries, in the common acceptation of that term. SCHOOTEN alſo, in his *Exercitationes Math.* (ſect. 24.) uſes the term Poriſm for an inveſtigation, without any preciſe object, of a variety of relations among lines drawn in and about a circle, or other geometrical figure. He gives only an example of a circle.

Bullialdus is the next author who mentions them in one of his *Exercitationes Geometricæ*, (1657,) but in it he refers to Fermat as the inventor, who had communicated the diſcovery in letters to ſome of his friends at Paris; Bullialdus, however, was unable to inveſtigate them.* This very eminent and ingenious man Fermat appears to have been the firſt in modern times, who made any near approach to the diſcovery of the Poriſms. He ſuppoſed that he had aſcertained the nature of theſe propoſitions, and that he had made ſuch progreſs as to enſure a compleat reſtoration of Euclid's work. In his treatiſe on the ſubject, which was publiſhed only after his death in 1679, he gives five propoſitions (without demonſtrations) as ſpecimens of his invention. Theſe are certainly Poriſms, but none of them belonged to Euclid's Treatiſe; and this circumſtance, beſides other arguments, juſtified Dr. Simson in ſtating that Fermat had not acquired a correct notion of the nature of a Poriſm.† It is ſufficient to remark that Fermat's definition of Poriſms

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* Dr. Simson, on the margin of his copy of this tract of Bullialdus wrote the following remark: "Ex hoc Tractatu liquet Bullialdum nihil de naturâ Poriſmatum intellexiſſe. R. S. Mart. 29, 1739."

† An Eloge on Monſieur Fermat appeared in the *Journal des Scavans*, Feb. 9, 1665, eight years after the publication of Bullialdus's tract, in which the ſuppoſed diſcovery of Fermat was announced to the world. This Eloge is alſo copied into the volume of poſthumous works publiſhed in 1679 by his ſon; but in the enumeration of his works which it contained, no mention is made of the Poriſms. Some readers may be gratified with Fermat's definition or deſcription of the Poriſms. "Cum Locum inveſtigamus Lineam rectam aut curvam inquirimus, nobis tantiſper "ignotum, donec Locum ipſum inveniendæ lineæ deſignaverimus, ſed cum ex "ſuppoſito Loco dato et cognito, alium Locum venamur, novus iſte Locus Poriſma "vocatur ab Euclide, qua ratione Locos ipſos unam ſpeciem, et eſſe et vocari, veriſſime Pappus ſubjunxit."—Fermatii Opera, vol. ii. p. 118.

(which at the ſame time is not very clearly expreſſed,) is avowedly derived from the definition of them by the Mathematicians reckoned modern, (νεώτεροι,) in the time of Pappus, and which he pointedly cenſures as inadequate. It may therefore be concluded from this authority, that Fermat's notion of a Poriſm was imperfect; more eſpecially as by this definition a numerous claſs of Poriſms, altogether unconnected with *Loci*, were neceſſarily excluded. The matter, however, is rendered perfectly clear by Dr. Simson's full expoſition of the nature of Poriſms; of which he gives a new definition, very different from that of Fermat, but fortified by accurate reaſoning, and the illuſtration of many examples, both of Euclid's Poriſms, and of others compoſed by himſelf and his friends. The Doctor alſo obſerves, that at an early period he had been ſtudying Fermat's Tract on Poriſms, and that in 1715 he had diſcovered the demonſtration of cas. 1. prop. 4. of Fermat's Propoſitions, before he knew any thing of the nature of Poriſms. So little could be derived from the labours of that diſtinguiſhed Geometrician, by Dr. Simson, in his inveſtigation of the common object of their purſuit.*

Dr. David Gregory alſo, an eminent Geometer in the latter part of the ſame century, ſeems, from having taken only a ſuperficial view of the ſubject, to have deceived himſelf, when he obſerves, in the preface to his valuable edition of Euclid, that it would not be difficult to reſtore the Poriſms of Euclid, if the Greek text of Pappus were publiſhed.

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* Dr. Halley, in the preface to the *Sectio Rationis,* (laſt page) ſpeaking of a work of Fermat's, adds, "qui (ſc. Fermatius) et Poriſmata Euclidis opus longe "difficillimum, redintegrare pollicitus eſt, verum fidem non liberavit."

That text was within his reach, as there were two Greek manuſcripts of PAPPUS in the Savilian Library, of which, as Profeſſor of Aſtronomy, he had a particular charge; but there is no evidence of his having made any attempt, and he died while engaged in another important work, the publication of the *Conics* of APOLLONIUS, which was finiſhed by his Colleague Dr. HALLEY. From the Savilian manuſcripts, Dr. HALLEY was enabled to give an improved edition of the preface to the ſeventh book of PAPPUS, with a translation much ſuperior to that of COMMANDINE. The general account of the Poriſms is in ſome places corrected; but the detail of the contents of EUCLID's work remained ſo very defective and unintelligible, that Dr. HALLEY, with his great abilities and learning, and with his ſucceſsful experience in reſtoring and elucidating ſeveral works of APOLLONIUS, was compelled to give up the inveſtigation of the Poriſms as hopeleſs; and he was brought to this concluſion, both from the difficulty of the ſubject, and alſo from the mutilations which the injuries of time had occaſioned in all the manuſcripts which had been examined.*

From all theſe ſeemingly unſurmountable obſtructions, it required an uncommon degree of zeal and fortitude in Dr. SIMSON to undertake the inveſtigation; and from his own account, which has already been given, his perſeverance was not leſs remarkable. From the time of his firſt diſcovery of the nature of a Poriſm, the Doctor, without neglecting his other ſcientific purſuits, occaſionally applied his mind to this very curious ſubject; and notwithſtanding the many difficulties which he encountered, he was at length completely ſucceſsful; not only in explaining the diſtinctive character of theſe Propoſitions, but in the inveſtigation of a number of the individual Poriſms of **Euclid**, and alſo of various other Poriſms, which were uſeful, both in illuſtrating the nature of theſe Propoſitions, and in ſhewing the application of them in the reſolution of difficult geometrical Problems.

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* See Note B. at the end.

After a certain progreſs in the proſecution of this ſubject, it became an important object to aſcertain a juſt definition of the Poriſm. The definition given by the later Mathematicians, as ſtated by **Pappus**, but cenſured by him, "quod "deficit hypotheſi a Theoremate Locali,"* clearly implies that a Poriſm had an immediate reference to a *Locus*; though it is not leſs clear that **Pappus** conſidered *Loci* as only one claſs of Poriſms, (a large one no doubt,) but that of courſe many Poriſms have no connection whatever with *Loci*.

But the definition which **Pappus** quotes from the ancients,† as more characteriſtic of Poriſms, is too general for any uſeful purpoſe; and though it does correſpond to the nature of theſe Propoſitions, yet it is deficient in diſcrimination, and of itſelf neither conveys any preciſe notion of **Euclid**'s Poriſms, nor gives aſſiſtance in the inveſtigation of any individual Propoſition.

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* "Quod deeſt in hypotheſi Theorematis Localis." **Hall.** verſio.

† **Pappus**, in his deſcription of the Poriſms, (preface to 7th book,) ſtates the following definitions of a Theorem, a Problem, and a Poriſm: "Differentias autem horum "trium, melius intellexiſſe veteres, manifeſtum eſt ex definitionibus. Dixerunt enim "*Theorema* eſſe quo aliquid proponitur demonſtrandum; *Problema* quo proponitur "aliquid conſtruendum; *Poriſma* vero eſſe quo aliquid proponitur inveſtigandum." **Hallei** verſio.

After much conſideration of various forms of a Definition which had occurred to him, the Doctor finally ſettled the following: "Poriſma eſt Propoſitio in qua proponitur demonſtrare rem aliquam, vel plures datas eſſe, cui, vel quibus, ut et "cuilibet ex rebus innumeris, non quidem datis, ſed quæ ad ea "quæ data ſunt eandem habent relationem, convenire oſtendendum eſt affectionem quandam communem in Propoſitione "deſcriptam."*

The Doctor illuſtrates the propriety and accuracy of this definition by many examples; and ſhews particularly wherein the definition blamed by Pappus coincides with his, and wherein it is deficient, by excluding many genuine Poriſms. The definition indeed, with much addreſs, is ſo framed as to correſpond with all the intimations of Pappus reſpecting Poriſms, and alſo with the character of the individual Poriſms of Euclid, which Dr. Simson had diſcovered; and therefore may juſtly be conſidered as expreſſive of the notions on this ſubject entertained by the ancients. It is not pretended that

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[*] From the neceſſary generality of expreſſion for comprehending every claſs of Poriſms, there is ſome obſcurity in this definition; but the Latin language, which Dr Simson uſed in all his mathematical writings, is in this caſe better fitted for giving preciſion, and for preventing ambiguity, than the Engliſh. In Mr. Lawson's tranſlation of Dr. Simson's Introduction to the Poriſms, which is plainly meant to be ſtrictly literal, this definition is expreſſed thus: "A Poriſm is a Propoſition in which it is propoſe "to demonſtrate that ſome one thing or more things are given, to which, as alſo to "each of innumerable other things, not indeed given, but which have the ſame "relation to thoſe which are given, it is to be ſhewn that there belongs ſome common "affection deſcribed in the Propoſition." The following leſs literal tranſlation is propoſed by Mr. Playfair, for remedying part of the obſcurity: "A Poriſm is a "Propoſition in which it is propoſed to demonſtrate that one or more things are "given, between which and every one of innumerable other things, not given, but "aſſumed according to a given law, a certain relation, deſcribed in the Propoſition, is "to be ſhewn to take place." Ed. Tranſact. vol. iii. p. 172, Note.

this was a definition of the ancients; for probably no preciſe definition was given by them, of either Theorem, Problem, or Poriſm. None appears in the works of the more early Geometers, which are ſtill preſerved in a conſiderable degree of purity, and where ſuch definitions would naturally have had a place. And we may affirm with much probability, that if any uſeful and characteriſtic definition of a Poriſm had reached the times of PAPPUS, he would not have neglected ſo valuable a remnant of ancient mathematical ſcience, in a work obviouſly deſigned for the preſervation of the more curious portions of it. He does not omit a definition, which probably was only a traditional and pointed obſervation of ſome ancient Geometer; and though of no uſe in explaining the character of a Poriſm, yet it in ſome degree fortified his objection to the definition of the later Mathematicians, who, he ſtates, from inability, could not accompliſh the inveſtigation of Poriſms; but ſatisfied themſelves with aſſuming the conſtructions as they found them in EUCLID, or other Geometers, and adding the demonſtrations.

It is obſerved by PAPPUS, that a Poriſm is neither a problem nor a theorem, but ſomething of an intermediate nature; and that it might be propoſed either as a problem or as a theorem; ſome Geometers contending for the one, and ſome for the other. Dr. SIMSON has given a form to the enunciation of a Poriſm, implying this intermediate character between a problem and a theorem. In his enunciation it is affirmed that certain things *may* be found, which ſhall have the relations or properties therein deſcribed. Perhaps this form reſembles more that of a theorem than of a problem; but at the ſame time, the things, of which it is ſaid that they may be found, muſt be actually inveſtigated by analyſis, as if the Propoſition were a problem. Were it ſimply propoſed to inveſtigate certain things which would have the properties expreſſed in the Poriſm, it may be regarded as a problem; but if theſe things are found by a conſtruction deſcribed in the enunciation, the Propoſition becomes a theorem, affirming the truth of the properties aſſerted; and then a demonſtration only is required, without any inveſtigation; in the manner which appears to have been practiſed by the later Mathematicians, alluded to by PAPPUS.*

The moſt ſatisfactory illuſtration of theſe definitions is by examples, of which Dr. SIMSON gives a great variety;

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* The enunciation of a Poriſm as a problem, is not conſiſtent with the uſual character of ſuch Propoſitions. Problems commonly (whatever difficulty attend the actual reſolution of them) are almoſt immediately recognized by thoſe having ſome knowledge of geometry, as either poſſible in certain circumſtances of the data, or as altogether impoſſible; and it is unuſual to propoſe as a problem, "to find things with "certain properties, reſpecting the poſſibility of which no judgment can be formed "without an analyſis, or ſuch conſideration as is equivalent to an analyſis." For example, if it had been propoſed as a problem in the time of APOLLONIUS, to find in a given parabola a point having the property of the focus, that point being then unknown; ſuch a Propoſition would not have been conſidered as a proper problem, but in reality would have been a genuine Poriſm.——PROCLUS, in his deſcription of Poriſms, mentions the 1 Prop. 3 Elem. "to find the center of a circle," as a Poriſm, being in ſome meaſure between a problem and a theorem. But in that caſe, as a circle, from EUCLID's definition of it, muſt have a center, the Propoſition "to find that center," ſeems to be a proper Problem. Had the circle been defined from another of its properties, as for inſtance, from its being produced by the extremity of a ſtraight line moving at right angles to another ſtraight line given in magnitude and poſition, and in the ſame plane, ſo that the ſquare of the moving line be equal to the rectangle by the ſegments into which it divides the given line; then the finding of the center would be a proper Poriſm, and might be enunciated thus: "within a given "circle (defined in the manner now mentioned) a point may be found, from which "all ſtraight lines drawn to the circumference will be equal."

and he diſtinguiſhes clearly the Locus and Local Theorem from the Poriſm, though all the three, in a large claſs of the latter, are convertible into each other. He proceeds to give examples of Poriſms altogether unconnected with *Loci*, and adds likewiſe an algebraical or arithmetical Poriſm; all of which, however, are clearly comprehended in his own definition, and diſplay the defect of the definition cenſured by Pappus.

After an ample expoſition of the nature of Poriſms, the Doctor proceeds to the reſtoration of ſome of Euclid's Poriſms, beginning with the general Propoſition contained in Pappus, though in an imperfect ſtate, and diſtributed into the ten caſes alſo remarked by him; adding the ſecond general Propoſition, which is an extenſion of the firſt. He then inveſtigates ſeveral other Poriſms aſcertained to belong to Euclid, from the remaining fragments of his deſcription; and by employing ſome of the *Lemmata* aſſumed by Euclid, and preſerved by Pappus, he proves the identity of his inveſtigation with that of the original author. The Doctor's inveſtigation of Poriſms, of courſe, is in the ancient analytic method; though from the general nature of theſe Propoſitions, ſome variation in the form, from what is uſed in a common problem, is requiſite. The things to be inveſtigated, as in the common analyſis, are ſuppoſed to be found; and the relation of them to the innumerable other things muſt alſo be aſſumed, and often, as exiſting in different ſituations, to expreſs effectually the general nature of the Propoſition.—It is unneceſſary here, however, to enter more particularly into this diſcuſſion, as the propriety and uſe of the form adopted by Dr. Simson can

can be fully explained only by examples, and will be intelligible to thoſe who have duly conſidered what he has written on the ſubject.*

FERMAT's five Propoſitions, though ſtated by him in the form of theorems, are undoubtedly Poriſms, and as no inveſtigation or demonſtration was given of them ſo far as Dr. SIMSON could learn, they are properly added. He gives them the appropriate form of Poriſms, renders them more general, and adds the neceſſary analyſis and compoſition. The ſecond Prop. of FERMAT belongs to the parabola, and is demonſtrated in the Doctor's *Conics*, Prop. 19. Lib. v. 2d edit.

The Poriſms of EUCLID, ſo far as may be conjectured from the deſcription of PAPPUS in its preſent imperfect ſtate, ſeem all to have had reference to the circle and ſtraight lines, and therefore may be called plane; but it is obvious that the Conic Sections may ſupply many ſolid Poriſms, and ſuperior lines in like manner may be the ſource of ſuperior claſſes of Poriſms, which the ancients would have called linear. Dr. SIMSON concludes this work with ſome elegant Poriſms of his own, and ſome by Dr. STEWART, one of the very few to whom he had, in the earlier part of his life, freely communicated his invention.‡ He takes occaſion alſo to ſhew the utility of Poriſms in producing eaſy reſolutions of geometrical problems, which, without that aſſiſtance, might be complex and difficult.

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* I ſhall quote here an important obſervation of Dr. SIMSON's on this ſubject. *Opera Reliq.* p. 337. "Semel igitur ad minimum, punctum, recta, angulus, vel "quodcunque fuerit de quo in Poriſmate aliquid generaliter affirmatur, ſumendum "eſt utcunque, intra tamen limites, ſi qui ſint in Poriſmate præſcriptos, ut legitime "fiat analyſis. Sæpe autem brevius et facilius ea quæ inveſtiganda ſunt invenientur, "ſi tum quævis ex prædictis rebus ſumatur utcunque, tum denuo modo quodam "particulari ſumatur, quando hoc fieri poteſt."

‡ See Note, 2 C. at end.

This application of Poriſms by the ancients is ſtated by Pappus; who inſinuates alſo, that little had been done reſpecting them by the geometers ſubſequent to Euclid; but as Euclid's treatiſe is part of the *τόϖος ἀναλυομένος*, we may preſume that this uſe of the Poriſms was in contemplation of the author,† and was probably regarded by ſubſequent geometers as the moſt important.

I cannot omit adverting in this place to a very ingenious theory of Poriſms propoſed by Mr. Profeſſor Playfair, of Edinburgh, firſt briefly in his account of the life of Dr. Stewart,* and afterwards more fully explained in a memoir on that ſubject in the third volume of the Tranſactions of the Royal Society of Edinburgh. The reſult of his inveſtigation is, that a Poriſm is the caſe of a problem which becomes indeterminate; or more particularly, "a Poriſm is a Propo-"ſition affirming the poſſibility of finding ſuch conditions "as will render a certain problem indeterminate, or capable "of innumerable ſolutions."§ But though I admire the ingenuity, and fully admit the ſoundneſs, of this definition, and alſo the utility of the principle on which it is founded, in the diſcovery of Poriſms, I muſt acknowledge my doubt of that particular notion of a Poriſm having ever been adopted, or even propoſed, among the ancient geometricians. The circumſtance of its being ſo ſatisfactory as a definition is to me a proof that it was never generally known or embraced: for had it ever been approved and eſtabliſhed, it ſeems ſcarce poſſible that it ſhould afterwards have been neglected and loſt. That among the ancients the conſideration of the relations ſubſiſting among the data, in ſome problems, might have occaſionally ſuggeſted the particular caſe in which theſe problems would become indeterminate, is very probable. It might alſo have often occurred to them, that this indeterminate caſe involved an important general Propoſition, which might be ſeparately ſtated as ſuch, and preſerved. Many Poriſms of EUCLID may poſſibly have been invented in that way; but ſtill I entertain a doubt, if ever the ancients were in poſſeſſion of this notion as a principle, and as the proper ground of the definition of a Poriſm. PAPPUS mentions the definition of the ancients, and apparently as the only one which they were known to poſſeſs, though, as has been remarked, it be of no particular uſe. He mentions alſo a definition of the later mathematicians, which he cenſures as erroneous: but if ſuch a compleat and ſatisfactory definition, which not only accurately diſtinguiſhes that claſs of Propoſitions, but points out an obvious ſource of the diſcovery of them, had ever been generally underſtood among the ancients, it is difficult to ſuppoſe that it could ever have been loſt; and had it reached the time of PAPPUS, it is moſt improbable that he ſhould neglect the recording of it in his detailed account of EUCLID's treatiſe on this ſubject. With theſe ſtrong internal probabilities, and the total want of external evidence, I muſt (with deference, however, to the opinion of thoſe who may think differently) adhere to the judgment which I have already expreſſed concerning the recent origin of this excellent definition propoſed by Mr. PLAYFAIR.*

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† It is proper to obſerve that Dr. Simson gives a corrected edition of all the *Lemmata* belonging to the Poriſms, which muſt be very uſeful to thoſe who may attempt to recover more of Euclid's Poriſms.

* This life of Dr. Stewart, read before the Edinburgh Society in July 1784, was publiſhed in the firſt volume of the Tranſactions.

§ In this paper is given a comprehenſive and very ingenious theory of the nature of Poriſms founded on this definition; to which the reader who is deſirous of ſatisfactory information on the ſubject is referred.

The very compleat account of the Poriſms given by Dr. SIMSON in this poſthumous treatiſe may be conſidered as ſo ſatisfactory an illuſtration of the nature and character of EUCLID's work, that the juſtice of the concluding remark of his preface will readily be admitted: "His igitur doc-"trinam Poriſmatum ſatis explicatam, et in poſterum ab "oblivione tutam fore, ſperare liceat." I may take the liberty of adding, that though Dr. SIMSON's fame, as an accurate, elegant, and ingenious Geometrician, be eſtabliſhed from his other works; yet the reſtoration of the Poriſms of EUCLID will be regarded by poſterity as the moſt important production of thoſe powers of inveſtigation and genius, with which he was ſo eminently endowed.

There remain ſome ſmall tracts in the poſthumous volume, which ſhall be very briefly mentioned.

The treatiſe *De Logarithmis* is formed on the model of the fifth book of EUCLID; and notwithſtanding the great variety of explanations which have been given of that celebrated and moſt uſeful invention, yet a deduction of the properties of *Logarithms*, with the ſame ſtrict manner of demonſtration which is uſed by EUCLID in treating of proportion, remained a deſideratum in the ſcience. It was undertaken at the requeſt of the late Earl STANHOPE, who ſuggeſted to him ſome valuable hints on the ſubject; and the tract, though ſhort, is compleat, and appears to be the laſt work which he finiſhed. His correſpondence with the Earl STANHOPE about the plan of it was in the year 1752; and the fair copy from which the tract was printed, is dated Feb. 19, 1762. But in this ſome corrections were made with the ſanction of the Earl STANHOPE.

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* See Note D. at the end. I may here obſerve that the two definitions are perfectly conſiſtent: but though Dr. SIMSON's might eaſily be derived from Mr. PLAYFAIR's, yet the deduction of Mr. PLAYFAIR's from the other is by no means ſo obvious, and does not appear to have ever occurred to Dr. SIMSON.

The next tract, *De Limitibus Quantitatum et Rationum*, is only a fragment; but it contains a rigorous demonſtration of the principles of fluxions, and of prime and ultimate ratios. It is well known that from the time of the invention of fluxions, ſome inaccurate expreſſions reſpecting infinitely little quantities, and ſome looſe reaſonings founded on ſuch expreſſions, had crept into that branch of ſcience. Much diſcuſſion aroſe, and a ſerious controverſy was maintained for ſome time; which, in a ſcience profeſſing to be founded on ſelf-evident principles, and conducted with ſtrict demonſtration, was of itſelf a matter of ſome reproach. Mathematicians of conſiderable name took a part in the buſineſs; and many treatiſes were written, with a view to remove the objections which had been ſtarted, and alſo to give a rigid demonſtration of the principles and rules of this new and moſt important analyſis. The doctrine of limits was certainly implied (though not in language calculated to prevent every objection) in Sir Isaac Newton's Propoſitions of prime and ultimate ratios. The principle, however, came afterwards to be ſtated in more accurate language; and is now by univerſal conſent admitted to be the ſound foundation on which this branch of the ſcience ought to reſt. This certainly has been properly explained in other treatiſes; yet it may be hoped that this fragment of Dr. Simson's will be conſidered by the admirers of the ſimplicity and accuracy of ancient geometrical reaſoning, not only as free from even the pretence of objection, but as a demonſtration, with elementary perſpicuity, of the principles of fluxions.*

At the end of the volume is an Appendix, containing a few geometrical problems reſolved in the ancient analytical method. This addition was made at the requeſt of the late Earl Stanhope; but it is proper to mention, that it does not appear that Dr. Simson ever intended to make any ſelection of ſuch problems, with a view to publication. There are, no doubt, among his papers, a great number of geometrical problems; ſome of which are valuable, but almoſt all of them appear to have been taken up only from the ſuggeſtion of the moment, either from his reading, his correſpondence, or from his own particular ſtudies; but generally without any deciſive marks of his having given a particular conſideration of the beſt poſſible ſolutions. The few added in this place were ſelected juſt when the volume was nearly printed, and without any particular object in view, except that examples might be given of the accurate determination of problems, about which the ancient Geometers were ſo very curious, and which has often been neglected by the moderns. I muſt obſerve, however, that the laſt problem, being a caſe of the *Tactions* of APOLLONIUS, will be mentioned more particularly afterwards.

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* It appears from Dr. Simson's papers, that he had been thinking of the method of limits as far back as 1736, and then propoſed to extend this method to the firſt and ſome other Propoſitions of the *Principia*. No trace however remains of his having compleated this deſign.

SECTION IV.

Of Dr. Simson's *Unpubliſhed Papers and Correſpondence.*

I SHALL conclude the account of Dr. Simson's labours and inventions with a few obſervations on his unpubliſhed papers, and his correſpondence.* From theſe papers it does not appear that he ever ſeriouſly embarked in the reſtoration of any other of the treatiſes in the *τόϖος ἀναλυομένος*, except thoſe already mentioned. In an early MS. volume, indeed, he gives an arrangement of the caſes of the Problem of the Tactions, and in other papers the ſolutions of moſt of them.†

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* Wherever in this Memoir I have made uſe of Dr. Simson's unpubliſhed papers, the reference to them is particularly remarked, that any inadvertence or miſtake of mine might not be attributed to him.

† The Tactions of Apollonius were firſt reſtored by Vieta, in a treatiſe, which he calls Apollonius *Gallus*. Afterwards they were reſtored by various mathematicians, both geometrically and algebraically. A Treatiſe of the Tactions by J. Gugl. Camerer was publiſhed at Gotha and Amſterdam in 1795, and is mentioned with commendation by Montucla, (tom. iii. p. 14,) but it contains only an edition of Vieta's treatiſe, with notes and additions, and a curious hiſtory of the Problem. The hiſtory is intereſting, from the accounts which it contains of the labours of ſome foreign mathematicians upon this problem, which are little known in this country. He gives the preface and *Lemmata* of the Tactions in Greek, with ſome various readings of ſeveral manuſcripts of Pappus. Though Vieta's ſolutions are elegant, yet they are in ſeveral reſpects deficient. There is not a full diſtinction either of the caſes, or of the neceſſary determinations. No analyſis is given, and no attempt to reſtore the Apollonian ſolutions by the uſe of the *Lemmata* in Pappus, which had been aſſumed in the work of Apollonius. See Camerer *de Tactionibus*, p. 4, and p. 12.

It would ſeem, however, that as there was little difficulty in this problem, it had not been ſufficiently intereſting to engage him in a compleat reſtoration of it. The uſe of one of the *Lemmata*, (Prop. 117, Lib. vii. PAPPI,)‡ in reſolving a caſe of the Tactions, did not for ſome time occur to him; but having diſcovered it, he deduced from it the elegant ſolution of that caſe which is printed in the poſthumous volume. I found among his papers the following date of this ſolution: " Feb. 9, " 1734, mane, poſt horam I^{mam} ante meridiem." So minute was he in ſome of theſe notices; and in this caſe perhaps, at the moment, he felt a little ſatisfaction from having overcome the only difficulty in reſtoring the Tactions in the APOLLONIAN method.

PAPPUS, in his deſcription of the Tactions, obſerves that there are other problems reſpecting Tactions, which were generally neglected by the editors; of whom, however, ſome prefixed one of theſe problems to the firſt of the two books of APOLLONIUS, as an eaſy and proper introduction to the doctrine of Tactions.* PAPPUS gives the general deſcription of this problem, from which Dr. SIMSON ſtated the arrangement and ſolution of the ſeveral caſes; and he alſo propoſed and reſolved another problem of Tactions, viz. " of points, lines, and " circles, any two being given to deſcribe a circle through " the points, or touching the ſtraight lines or circles, of which

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‡ See Note E. at the end.

* This Problem is " of points, ſtraight lines, and circles, given in poſition any two, " to deſcribe a circle given in magnitude, which may paſs through the given point " or points, and touch alſo (if poſſible) the given lines or line." This problem was " alſo reſolved by MARINUS GHETALDUS.

" the centre ſhall be in a ſtraight line given in poſition." But all theſe problems, though very eaſy, may be occaſionally uſeful, when a propoſed problem can be reduced to a caſe of any of them.

With reſpect to the *Inclinations* of Apollonius, he ſeems to have beſtowed little attention; only it may be obſerved, that, after repeated unſucceſsful trials, he found (February 20, 1723) the ſolution of the problem about the Rhombus, by the Lemma of Pappus, (Prop. 70, lib. vii.) which Apollonius no doubt had employed in his ſolution. The figure of this Lemma in Commandine's Pappus, which is erroneous, was at the ſame time amended.* The *Inclinations* had been reſtored by Marinus Ghetaldus ; and ſubſequently to Dr. Simson's time, by Dr. Horsley, the late Biſhop of St. Aſaph, in 1772; and alſo by Mr. Wales, in 1779. I may further remark, that Dr. Simson made ſome uſeful corrections of another Lemma of the *Inclinations*, viz. Prop. 85, lib. vii. Pappi. And in this Propoſition is an example of the deduction of a problem to a caſe of the *Sectio Determinata*, which is thence conſidered by Pappus as compleatly reſolved; and in the demonſtration a reference is made to that caſe, being *Epitagma* 2. Prob. 3. lib. i. *Sect. Determ.* In Commandine's time the loſt treatiſe *De Sect. Determ.* had not been reſtored; and therefore, in his commentary on this Propoſition, he gives a ſolution of that caſe, as neceſſary to the ſolution of this lemmatic problem.

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* See Note F. at the end, in which is given Dr. Simson's ſolution, and the amended figure of the Lemma. It may be obſerved, that an ingenious correction of the Lemma, and of its figure, as it is delineated in the Savil. MS. No. 3, is given by Dr. Horsley, in his *Reſtitution of the Inclinations.*

Among the Doctor's papers there remain alſo ſome fragments reſpecting *Loci ad ſuperficiem*, of which EUCLID compoſed two books, making a part of the τόϖος ἀναλυομένος. PAPPUS, in his preface to his ſeventh book, gives no deſcription of theſe two books, nor any account of their contents, from which, as in other caſes, geometers might have been enabled to explain or reſtore them. There are, indeed, a few *Lemmata* at the end of his ſeventh book, ſaid there to belong to the *Loci ad ſuperficiem*, but the firſt, as it is enunciated, and the other four,[*]

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[*] Theſe five *Lemmata* are in COMMANDINE's tranſlation, and ſome of the MSS. in a very imperfect ſtate, as will appear from the following ſhort account of them.

The enunciation of Lemma 1, (Prop. 235.) is a ſimple Conic *Locus,* but in a ſort of ſupplementary enunciation is an alluſion to a *Locus ad superficiem,* which in its preſent ſtate is unintelligible; but a conjecture about the meaning of it is propoſed by Dr. WALLIS, in the Savilian copy of COMMANDINE.

In fol. 300. a. COMMAND. 1588, the four laſt lines, though in the type of the commentary, is a part of the text, and the Greek, of which it is a verſion, is in the Savil. MS. No. 3. It is the enunciation of a well-known Conic *Locus,* but without any demonſtration; and at the end it is ſaid, " that it will be ſhewn by premiſing the " following *Locus.*" This probably was Lem. 2, as the next Propoſition 236 (viz. the *Locus* referred to) is Lem. 3.

Prop. 236. Lem. 3. is alſo a well-known Conic *Locus,* conſiſting of three caſes, belong to the three ſections; but the firſt caſe of the parabola is alone demonſtrated in this Propoſition.

Prop. 237. Lem. 4. This is the enunciation of the two caſes omitted in the preceding Prop. 236; viz. thoſe of the ellipſe and hyperbola, and theſe are analyzed and demonſtrated.

Prop. 238. Lem. 5. This propoſition is only a repetition of what I ſuppoſe to have been Lem. 2, and is the ſame *Locus* to the three ſections as was there ſtated. PAPPUS demonſtrates only the firſt caſe of the parabola, by a reference to Prop. 236; but the other two caſes of the ellipſe and hyperbola are eaſily demonſtrated by a ſimilar reference to Prop. 237.

I may further remark that the *Locus ad hyperbolem,* in Prop. 237, is aſſumed by PAPPUS in the ſolution of a ſolid problem, Prop. 34, b. iv. without however quoting either Prop. 237 of b. vii. or APOLLONIUS. And no doubt it was at the time a well-known ſolid *Locus.*

are all ſolid *Loci*; and truly contain only two *Loci*, of which the one is preliminary to the other. What follows the enunciation of the firſt (viz. Prop. 235) in its preſent mutilated ſtate is unintelligible; it has, however, ſome alluſion to a *Locus ad ſuperficiem*, but the other four have none whatever. It is indeed ſomewhat ſingular, that theſe Propoſitions are introduced in this place. PAPPUS, in his preface, profeſſes to give an account of the treatiſes of the *τόϖος ἀναλυομένος*, only ſo far as the *Conics* of APOLLONIUS, in the arrangement aſſumed by him, which is not the order of time, excluding thereby (from deſign, we may fairly ſuppoſe) the *Loci ad ſuperficiem*, and ſeveral others. Perhaps theſe *Lemmata* may be a fragment of ſome other work, as probably is the Propoſition, called λήμμα τȣ͂ ἀναλυομένȣ τόϖȣ, ſubjoined to them in ſeveral MSS. which has no obvious connection with any other Propoſitions of that ſeventh book. It is poſſible, however, that theſe five Propoſitions may have been *Lemmata in Locos ad ſuperficiem*; but being ſolid *Loci*, might have been conſidered as a proper addition to the "*Lemmata in Conica* APOLLONII," to which they are ſubjoined.

In the deſcription of the *Loci Plani*, however, a ſhort account is given of the *Loci ad ſuperficiem*, and they are mentioned alſo in Prop. 28 and 29, lib. iv. PAPPI; which are problems reſolved by means of *Loci ad ſuperficiem*, and which Dr. SIMSON has corrected and rendered intelligible. Theſe are all the materials remaining for the inveſtigation of this difficult and curious, though perhaps not very important, piece of ancient geometry.

Dr. Simson had begun very early to conſider theſe *Loci*; for in 1721 I find ſome equations for expreſſing the relation of the co-ordinates, by means of three indeterminate quantities. It appears alſo that he had afterwards deſigned to give a regular explanation of theſe *Loci*; for there remains a preface for ſuch an eſſay, in which he propoſes to explain the method of inveſtigating ſuch *Loci* by examples. From theſe he hopes that it would be eaſy for one well acquainted with ſolid *Loci* to inveſtigate and deſcribe any *Loci ad ſuperficiem* which can ariſe from Conic Sections variouſly ſituated; ("*vario ſitu diſpoſitis.*") In the beginning of the preface, where he remarks the very little known of theſe *Loci* among the moderns, he has a marginal correction, intimating that he had not, at the time of writing this preface, ſeen the treatiſe *Des Courbes a double Corbure*, which he aſcribes (ſurely from inadvertence) to D. N.* But as there is no treatiſe of that period of this title, or indeed on this ſubject, but that of M. Clairaut, publiſhed in 1730, there can be little doubt of that being the work alluded to by Dr. Simson.†

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* M. Nicole, in a Memoir of the Academy of Sciences, (1734) but ſubſequent to the publication of Clairaut's work, makes an alluſion to the curves of double curvature. M. Nicole was one of the cenſors of Clairaut's treatiſe.

† The very ſlight notice of the ancient *Loci ad ſuperficiem*, by Des Cartes, in his Geometry, (book ii.) was of no uſe for Dr. Simson's inveſtigation. "At vero duabus conditionibus deficientibus ad hujus puncti determinationem, Locus in quo illud reperitur ſuperficies eſt," &c. This, to be ſure, correſponds to an equation with three unknown quantities, but it is not purſued by Des Cartes. His commentator Schooten gives two examples, which may perhaps be included in the general deſcription of the *Loci ad ſuperficiem* by Pappus, in his account of *Loci Plani*; but the Data and the Locus are all in the ſame plane, and they furniſh no illuſtration of what Dr. Simson conſiders as the object of the ancient treatiſe of Euclid.——See Cartesii *Geomet.* Amſt. 1683, p. 34; and Schoot. *Comment.* p. 228. See alſo Schootenii *Sectiones Miſcellaneæ*. Lugd. Bat. 1657, p. 494.

All theſe are comprehended in the following equation:

$$xx+\frac{a}{b}xy+ax+by+\frac{a}{c}yy+\frac{a}{d}yz+\frac{a}{e}xz+gz+\frac{h}{k}zz = \text{Dato.}$$

The ſigns being any how changed.*

In proceeding however to examples, he treats the ſubject geometrically, and without any reference to the equation. There are indeed ſeveral examples, but all of them, except a few of the ſimpleſt forms, are unfiniſhed; and though in the firſt there is a conſiderable progreſs, and an inveſtigation of the common ſection of the *Locus ad ſuperficiem* ſought, when cut by ſeveral planes in different poſitions, yet he mentions ſeveral others to be aſcertained, but which are not further proſecuted. The Doctor was not eaſily diverted from an inveſtigation, merely from the difficulty of it; and we may therefore reaſonably infer, that he abandoned the ſubject, partly from its having been previouſly taken up by CLAIRAUT, and partly from conſidering it as a branch of ancient geometry, of inferior importance and utility.† The omiſſion of any deſcription of this and ſome other treatiſes of the *τόϖος ἀναλυομένος*, in the preface to the ſeventh book, is a preſumption of a ſimilar eſtimation of the *Loci ad ſuperficiem* being entertained by PAPPUS.‡

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* See Note G.

† See Note G. at the end, ſome of the eaſieſt examples.

‡ In vol. ii. p. 151, of FERMAT's works, there is a letter of his to ROBERVAL, in which, after ſome obſervations about *Loci*, and particularly reſpecting one of the ſecond book of APOLLONIUS, he adds, "*de Locis ſolidis et ad ſuperficiem multa quoque* "*ſunt detecta.*" He ſlightly alludes to the ſame ſubject in a letter to Sir KENELM DIGBY. WALLISII *Opera*, tom. ii. p. 859. Alſo in page 116 of ſecond vol. of FERMAT's works, in the preface to his Poriſms, after mentioning the reſtoration of ancient treatiſes of geometry by SNELLIUS, VIETA, and MARINUS GHETALDUS, he adds, "ſequebantur *Loci plani*, *Loci ſolidi*, et *Loci ad ſuperficiem*, et huic quoque

Beſides theſe fragments of geometrical ſpeculations, there are a great variety of Problems, Theorems, Loci, Data, and Poriſms, diſperſed through the Doctor's papers, and particularly in the MS. volumes, which he called *Adverſaria*, of which eighteen remain. Theſe in general, as has already been obſerved, appear to have been analyzed or demonſtrated, as the circumſtances of his reading, converſation, or ſtudy, happened to ſuggeſt; yet they may be conſidered as a valuable repoſitory of ſuch Propoſitions, from which uſeful ſelections might be made, for promoting the ſtudy of geometry in the ancient method. They are depoſited in the College library of Glaſgow; and it is much to be wiſhed, that ſome gentleman acquainted with the ſubject, and who has leiſure and inclination for the taſk, would take the trouble of arranging a proper collection.*

It has been repeatedly remarked, that notwithſtanding Dr. SIMSON's early and continued partiality for geometry as

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" parti non ignoti nominis geometræ, ſuccurrerant, eorumque opera manuſcripta, " licet et adhuc inedita, latere non potuerint." And in another letter to Mr. ROBERVAL (p. 153) he obſerves, " J'en ay plus de cens Propoſitions tres belles et " particulierement des *Lieux solides* et *ad superficiem*, mais le loiſir me manque, &c." No further account however of theſe reſtorations of *Loci ad superficiem* is to be found.

* There are in theſe volumes, alſo, a number of geometrical demonſtrations, by the Doctor, of theorems, in the works of various eminent analyſts, where they are treated either algebraically, or if geometrically, with inferior ſkill.—I may remark that moſt of the Propoſitions alluded to are dated; and on ſome particular occaſions not only the day, but even the hour is noted. Sometimes alſo, when he had accompliſhed a ſolution or a demonſtration, which pleaſed at the moment, ſome expreſſion of his ſatisfaction is added. He began this practice very early, even before he was elected Profeſſor of Mathematics, and it was continued till about fourteen months before his death. In the book of *Adverſaria* Q. is the lateſt date now to be found, 11th Auguſt 1767; he being then in his eightieth year.

cultivated by the ancients, he did not entirely neglect the modern analyſis, but became well acquainted with algebra, and with the common applications of it which were purſued in the early part of the laſt century.* He certainly did not approve of the prevailing uſe of algebra in geometrical inveſtigations, which could be accompliſhed with more elegance, and not ſeldom with more eaſe, by the ancient method; but he made no ſcruple of acknowledging the ſuperior power of algebra in complex geometrical inquiries, and more eſpecially in the application of it to ſuch as are phyſical.—It is probable however, from all the information which can now be obtained reſpecting his ſtudies, that though he underſtood the modern analyſis as delivered by Sir ISAAC NEWTON, yet he did not continue to devote much of his time to the ſtudy of it, and thence did not enter much into the profound analytical ſpeculations of ſucceeding mathematicians.

It muſt be admitted, alſo, that his rigid notions reſpecting algebraical reaſoning, particularly in geometry, had a tendency to limit inveſtigations, by that method; and to diſcourage him from purſuing the methods of modern analyſts, who felt no ſuch difficulties as had occurred to him. As an example, however, of Dr. SIMSON's knowledge of algebra, I may mention his invention of ſome very ſimple and excellent ſerieſes for finding the circumference of a circle, of which he appears to have given an intimation in 1722 to Dr. JURIN, then ſecretary of the Royal Society.—Dr. JURIN undertook to obtain the ſentiments of his learned friends on theſe ſerieſes; and therefore Dr. SIMSON, in a letter of Feb. 1, 1723, tranſmitted to him his paper on the ſubject,* containing the ſerieſes, with two elementary Propoſitions from which they were derived. Though Dr. JURIN was not apprized that any thing of the kind had been done by others, yet by inquiry he found that Mr. MACHIN, near twenty years before, had communicated to the Royal Society ſome ſerieſes of the ſame form, but had afterwards withdrawn them. Mr. MACHIN readily gave a copy of his ſerieſes to be communicated to Dr. SIMSON, which Dr. JURIN ſent to him immediately after; (March

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* Dr. SIMSON often amuſed himſelf, eſpecially in the latter part of his life, in reſolving any geometrical problems which attracted his notice, by the algebraical method, though he rarely made any entry of theſe ſolutions in his *Adverſaria*.

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* I annex a copy of the portion of this letter which relates to the ſerieſes. The remainder of it reſpects the Poriſms, and is taken notice of in another place. For the paper encloſed, ſee Note H. at the end.

"SIR,

"Friday laſt I had the favour of your very civil anſwer to the letter I lately gave "you the trouble of: ſome private affairs have kept me from tranſcribing the "encloſed trifle, elſe you ſhould have had it ſooner.—The ſeries for finding the "circumference of the circle which are in Mr. SHERWIN's Book of Tables, being "deduced from tangents of arches which have a given ratio to the circumference, I "was deſirous to try if the like might not in a general way be drawn from tangents, "whoſe arches are not in a given ratio to it; and found a great many eaſily flowed "from the two Propoſitions and their corollaries, which I have encloſed. The Pro"poſitions, though obvious enough, I do not find in Mr. BRIGGS, or any of the authors "I have ſearched for them: the corollaries are long ſince known, if I miſtake not. "Among the ſeries I have ſet down, there is one which Mr. JONES, in his Synopſis, "ſays he had from Mr. MACHIN, and which is certainly one of the beſt that can be "found by this or any other method. Whether any or all of the reſt have been taken "notice of by any body, I am wholly ignorant; but you may ſoon diſcover.

"I thankfully acknowledge the favour of your kind offer to obtain the ſentiments "of ſome of your learned friends upon the encloſed paper, but I entreat you may "give yourſelf no more trouble about it, ſince I am aſhamed to have given you "already ſo much, when the thing is ſcarce worth taking notice of, and which you "will eaſily ſee is not to be ſhewn to any but as ſuch."

"Glaſgow, Feb. 1, 1723."

1723). The extreme modeſty with which Dr. SIMSON mentions this ingenious inveſtigation to Dr. JURIN, is remarkable; for though from the preſent ſtate of ſcience, thoſe serieſes may have loſt their former importance, yet near a century ago they had the merit of novelty and utility, and had then occupied the attention of Mr. MACHIN and Dr. SIMSON. It may thence be ſatisfactory to ſome readers to ſee a fuller account of them, which is given in a note at the end.*

The Mathematicians of the end of the ſixteenth and beginning of the ſeventeenth centuries began to apply algebra to geometrical inquiries; and this method was much promoted and extended by DES CARTES. The uſe of negative and impoſſible roots of equations was alſo introduced into this analyſis; though by preceding algebraiſts they were at firſt not obſerved, and afterwards when obſerved were reckoned uſeleſs. While algebra, in its early and humble ſtate, was employed almoſt entirely about numbers, theſe roots were naturally neglected, and the language of the firſt writers on the ſcience was thence not encumbered with the metaphyſical difficulties which aroſe from the proceſſes and reaſonings of the followers of DES CARTES. Some of theſe were reaſonably conſidered by Dr. SIMSON as defective in that preciſion of definition, and ſtrictneſs of argument, which have ever been the boaſt of pure geometry, and to which he had been accuſtomed from his almoſt conſtant ſtudy of that branch of the ſcience. The Doctor, from this cauſe perhaps, conceived a prejudice againſt an application of algebra, which was accompanied with ſuch difficulties; and was thence led to treat of that ſcience after the manner of the early writers on it, with the limited definition and uſe of the negative ſign. Many detached obſervations on this ſubject remain among his papers;* and ſome ſhort eſſays alſo on cubic equations, in which he endeavours to explain them without admitting negative or impoſſible roots. In the preſent ſtate of that ſcience, it is unneceſſary to mention the detail of theſe ſpeculations; and it may be ſufficient to remark, that though the method of inveſtigation was different, the reſults were ſimilar to thoſe contained in the Eſſay on the Negative Sign by that learned promoter and patron of mathematical ſcience, Mr. BARON MASERES; of which, from the correſpondence of opinions, Dr. SIMSON ſpoke with much approbation. Dr. SIMSON's notions on this ſubject probably varied a little in the progreſs of life, by his confirmed partiality for the ancient analyſis, and an encreaſing prejudice againſt the algebraical. The elegance and ſatisfactory clearneſs of every ſtep in the geometrical method muſt, where it can be employed, be univerſally preferred to the almoſt mechanical proceſs of an algebraical calculation. His animadverſions, however, on the application of algebra to geometry, chiefly referred to thoſe caſes where it was not neceſſary, and in which the more excellent method of the ancients could be ſucceſsfully employed. In Plane Geometry, and in the Conic Sections, the ancient method is ſtill generally uſed, and by good judges is allowed to be the beſt for forming the taſte, and exerciſing the reaſoning powers, of young perſons; and Dr. SIMSON has done eſſential ſervice by his geometrical works, in exciting curioſity reſpecting theſe branches, and facilitating the ſtudy of them.

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* See Note H.

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* See in Note K. at the end, ſome obſervations by Dr. SIMSON on this ſubject, in his letter to Mr. GEO. LEWIS SCOTT.

Dr. SIMSON has no where in his writings given any ſpecific opinion of the proper and neceſſary uſe of algebra in geometrical enquiries, and therefore we cannot judge with preciſion of his views reſpecting it. It may be obſerved, however, that as the indefinite number of claſſes of geometrical curves, in the preſent ſtate of both the ancient and modern analyſis, can be conveniently treated of only by algebra, this application of algebra muſt be admitted as neceſſary; and the definition of the Conic Sections by equations may be regarded as an uſeful introduction to the definition of ſuperior curves on that principle. Dr. SIMSON ſeems, in the early part of his life at leaſt, in ſome meaſure to have admitted this; for in a great number of ſolid *Loci* among his papers, though demonſtrated geometrically, the ſuitable equation is added. Probably, however, he never ſet about taking an accurate and comprehenſive view of the ſubject in all its bearings; and having made his early choice from taſte, the ancient analyſis continued through life to be the favourite object of his ſtudy.

Conſidering the unbounded extent of mathematical ſcience, and the variety of branches into which it may, and has been uſually diſtributed; it may be for the advantage of the moſt ſuccefsful progreſs of the whole, that individual mathematicians ſhould apply themſelves chiefly to particular diviſions of it. Though there be a cloſe connection among theſe ſeveral branches, yet a moderate knowledge of the general ſyſtem may be ſufficient for the ſucceſsful cultivation of any one portion of it, to which the attention of a mathematician may happen to be directed. Had Dr. Simson applied his ardent mind to the modern analyſis, moſt probably the reſults of his inveſtigations would have been important; but his not having devoted much of his attention to the ſtudy of it, was certainly favourable to his progreſs in the laborious reſearches which he undertook reſpecting the ancient geometry. It will be readily admitted alſo, that if ſome of the great modern analyſts happen not to have ſtudied the ancient geometry, as may be preſumed to be the caſe, they were not thereby materially obſtructed in their profound inveſtigations, by which human knowledge has been ſo much enlarged; though, perhaps, by ſuch previous ſtudy, their works might in ſome inſtances have been improved in ſimplicity and perſpicuity.

It appears from the Doctor's letters to the late Earl Stanhope,* that his Lordſhip had recommended to him to attempt the demonſtration of ſome of the more important modern geometrical diſcoveries by the ſtrict and more elegant method of the ancients. This was in 1751, and though he ſtates it to be very much his own wiſh to make the trial, he conſiders it to be beyond his power, from the decline of his faculties. The tract on *Logarithms*, and the fragment *De Limitibus*, ſeem to be his only attempts of that kind; but he propoſes to recommend it to his ſcholars, Dr. Stewart, Dr. Moor, and Mr. Williamson. There are indeed among his papers a few imperfect ſketches (apparently of an old date) of attempts to demonſtrate geometrically the celebrated theorem of the Circle invented by Mr. Cotes. He does not ſeem to have beſtowed much time to this ſubject; but I have heard him mention that his plan was to demonſtrate firſt the more eaſy caſes, and then to attempt to frame a general method of deducing any caſe of this Propoſition from the next preceding and more ſimple caſe, from which a compleat demonſtration of the theorem in its full extent might be derived. But in this he was unſucceſsful, and he regretted the diſappointment. But as neither Dr. Simson, nor any other geometer, ſo far as is known, have ever accompliſhed a geometrical demonſtration of this theorem, we may conſider the inveſtigation of Mr. Cotes as an important example of the ſuperior power of the modern analyſis, in the diſcovery and demonſtration of a Propoſition, very general and difficult indeed, but ſtrictly geometrical; and though it be commonly expreſſed algebraically, yet may, by means of compound ratios, be enunciated in the pure language of geometry.

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* See Note I. at the end.

Long before any ſymptom of decline in his health appeared, he ſeems to have felt ſome remarkable decay of his memory and attention. This was not obſerved by his friends, either in converſation or in correſpondence; but his complaints of it were ſo particular and ſerious, that no doubt can be entertained of the fact, nor of the influence which it had in diverting his mind from ſome of the more ſevere inveſtigations which he had formerly wiſhed to purſue.*

In the preface to the Doctor's *Conic Sections*, when recommending the geometrical method of treating the ſubject, in preference to the algebraical, he has this remarkable ſentence: "In quibus autem differat analyſis geometrica ab ea quæ "calculo inſtituitur algebraica, atque ubi hæc aut illa ſunt "uſurpanda, atque quæ ſint in mathematicis utriuſque partes "propriæ, alias differendum." From this obſervation it muſt be acknowledged, there naturally ariſes a preſumption, which has been generally entertained, that the Doctor had conſidered the ſubject, and probably written ſomething concerning it: and the opinion of ſo diſtinguiſhed a mathematician on a point of ſome difficulty, and of which he was well qualified to judge, would have been particularly intereſting. But among his papers no trace of ſuch an attempt is to be found. Profeſſor Robison, in his Life of Dr. Simson already mentioned, ſeems to think that a diſſertation on *Loci*, which he had ſeen when attending the Doctor's Lectures in the College of Glaſgow, might have been the tract to which the preceding quotation referred.* Of this, however, notwithſtanding the weight of Mr. Robison's opinion, a reaſonable doubt may be entertained, conſidering the extent of the diſcuſſion implied in that quotation, which muſt have gone far beyond what ſuch a diſſertation as Mr. Robison mentions, could be ſuppoſed to contain. But Dr. Simson's own teſtimony, in his correſpondence with Mr. Scott reſpecting this very ſentence, is concluſive, and puts an end to all ſpeculation regarding it, as he therein explicitly declares, that on the ſubject of it "he had no "papers, and never wrote any thing on that matter." Fortunately, a conſiderable part of this correſpondence is preſerved; and the moſt important portion of it is placed in a note at the end.* The letters both of Mr. Scott and Dr. Simson will be read with intereſt: they contain ſome valuable obſervations on the relative merits of the ancient analyſis and of the algebraical, in the ſolution of geometrical problems. The ſubject indeed is not exhauſted; and as the correſpondence took place from occaſional circumſtances, the diſcuſſion, though entertaining and inſtructive, is of courſe limited very much to the points which thus accidentally aroſe. It touches little on the peculiar powers of the modern analyſis, and is confined chiefly to the compariſon of the two methods in caſes where either may be employed.†

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* See Note K. at the end.

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* No trace of this diſſertation on *Loci* remained among his papers.

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* See Note K.

† For ſome remarks on this correſpondence, ſee the latter part of Note K.

SECTION V.

Sketch of Dr. SIMSON'*s Character.*

IT was formerly obſerved that Dr. SIMSON's mathematical labours and inventions, of which ſome account has been given in this memoir, were the moſt intereſting occurrences of his life, and moſt illuſtrative of his capacity and genius.

In attempting, however, to give a brief outline of the Doctor's character, beſides the talents diſplayed in his geometrical works, which claim the firſt conſideration, it may be expected that ſome obſervation alſo ſhould be made of the well-known diſcriminative features of his mind, which, though not important in themſelves, appear to have had an influence on its greater qualities. Theſe peculiar features are ſtill affectionately remembered by his pupils, friends, and acquaintances who ſurvive; and, from the eminence of his genius, they may be regarded by others as objects of a natural and liberal curioſity.

Dr. SIMSON was originally poſſeſſed of great intellectual powers, an accurate and diſtinguiſhing underſtanding, an inventive genius, and a retentive memory: and theſe powers, being excited by an ardent curioſity, produced a ſingular capacity for inveſtigating the truths of mathematical ſcience. By ſuch talents, with a correct taſte, formed by the ſtudy of the Greek Geometers, he was alſo peculiarly qualified for communicating his knowledge, both in his lectures and in his writings, with perſpicuity and elegance. He was at the ſame time modeſt and unaſſuming; and though not indifferent to literary fame, he was cautious and even reſerved in bringing forward his own diſcoveries, but always ready to do juſtice to the merits and inventions of others. Though his powers of inveſtigation in the early part of life were admirable, yet before there appeared any decline of his health, he felt ſtrong impreſſions of the decay both of his memory and other faculties; occaſioned probably by the continued exertion of his mind in thoſe ſevere ſtudies, which for a number of years he purſued with unremitting ardour.

Beſides his mathematical attainments, from his liberal education he acquired a conſiderable knowledge of other ſciences, which he preſerved through life, by occaſional reading, and in ſome degree alſo, by his conſtant intercourſe with ſo many literary men in his College.† He was eſteemed a good claſſical ſcholar, and though the ſimplicity of geometrical demonſtration does not admit of much variety of ſtile, yet in his works a good taſte in that point may be diſtinguiſhed. In his Latin prefaces alſo, in which there is ſome hiſtory and diſcuſſion, the purity of language has been generally approved. It is to be regretted indeed that he had not had an opportunitity of employing, in early life, his mathematical and Greek learning, in giving an edition of Pappus in the original language.[‡]

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† Dr. Simson in the early part of his life ſtudied Botany, in which he made great progreſs, and it became a ſource of amuſement to him in his walks. In the year 1746, when the Univerſity of St. Andrew's wiſhed to confer on him the degree of Doctor, as he was a layman, a degree of medicine was propoſed from the circumſtance juſt mentioned, though he had no other pretenſion to diſtinction in that ſcience.

It may be mentioned alſo in this place, that a few years after he became a Profeſſor, he compoſed a tract on the School Logic, with rigorous demonſtrations of its rules. Some ſketches of it remain, which ſtrongly mark the accuracy of his reaſoning powers to whatever ſubject they were directed.

Dr. Simson never was married; and the uniform regularity of a long life, ſpent within the walls of his College, naturally produced fixed and peculiar habits, which however, with the ſincerity of his manners, were unoffending, and became even intereſting to thoſe with whom he lived. The ſtrictneſs of theſe habits, which indeed pervaded all his occupations, probably had an influence alſo on the direction and ſucceſs of ſome of his ſcientific purſuits. His hours of ſtudy, of amuſement, and of exerciſe, were all regulated with uniform preciſion. The walks even in the ſquares or garden of the College were all meaſured by his ſteps, and he took his exerciſe by the hundreds of paces, according to his time or inclination.

[‡] Dr. Simson's learned diſcuſſion concerning ſome Greek terms in the account of the Poriſms by Pappus has already been mentioned, and a copy of it is inſerted in Note C. I may mention here alſo another example of his critical knowledge. In the dialogue of Plato, Meno, there is a paſſage, of which, even in the beſt editions, ſome ſentences are ſomewhat obſcure, and probably corrupted. It relates to an ingenious ſpeculation, of all knowledge being only reminiſcence; and as an illuſtration, an uninſtructed youth is led, merely by an examination, to aſſent to the truth of a geometrical propoſition, (an eaſy caſe of the 47. 1. Elem. Eucl.) This had attracted Dr. Simson's notice; and he found that ſome natural and eaſy emendations, with the addition of a diagram, which, in the dialogue, is plainly ſuppoſed to be exhibited to the youth, rendered the argument intelligible and correct. He had drawn up a detailed ſtatement of the paſſage, and the neceſſary corrections which, in his own time I ſaw in the hands of one of his colleagues, who has been dead many years: but no trace of it is now to be found among the Doctor's papers, or any where elſe.

It has been repeatedly mentioned, that an ardent curioſity was an important feature of his character. It contributed eſſentially to his ſucceſs in his mathematical inveſtigations, and it diſplayed itſelf in the ſmall and even trifling occurrences of common life. Almoſt every object and occurrence excited it, and ſuggeſted ſome problem which he was impatient to reſolve. This diſpoſition, when oppoſed, as it often neceſſarily was, to his natural modeſty, and to the formal civility of his manners, occaſionally produced an embarraſſment, which was amuſing to his friends, and ſometimes a little diſtreſſing to himſelf.

The Doctor, in his diſpoſition, was both cheerful and ſocial; and his converſation, when he was at eaſe among his friends, was animated and various, enriched with much anecdote, eſpecially of the literary kind, but always unaffected. It was enlivened alſo by a certain degree of natural humour; and even the ſlight fits of abſence to which in company he was occaſionally liable, contributed to the entertainment of his friends, without diminiſhing their affection and reſpect, which his excellent qualities were calculated to inſpire. One evening in the week he devoted to a club, chiefly of his own ſelection, which met in a tavern near the College. The firſt part of the evening was employed in playing the game of whiſt, of which he was particularly fond; but though he took no ſmall trouble in eſtimating chances, it was remarked that he was often unſucceſsful. The reſt of the evening was ſpent in cheerful converſation, and as he had ſome taſte for muſic, he did not ſcruple to amuſe his party with a ſong; and it is ſaid that he was rather fond of ſinging ſome Greek odes, to which modern muſic had been adapted. On Saturdays he uſually dined in the village of

Anderſton, then about a mile diſtant from Glaſgow, with ſome of the members of his regular club, and with a variety of other reſpectable viſitors, who wiſhed to cultivate the acquaintance, and enjoy the ſociety, of ſo eminent a perſon. In the progreſs of time, from his age and character, it became the wiſh of his company that every thing in theſe meetings ſhould be directed by him; and though his authority, growing with his years, was ſomewhat abſolute, yet the good-humour with which it was adminiſtered, rendered it pleaſing to every body. He had his own chair and place at table; he gave inſtructions about the entertainment, regulated the time of breaking up, and adjuſted the expenſe. Theſe parties, in the years of his ſevere ſtudy, were a deſirable and uſeful relaxation to his mind, and they continued to amuſe him till within a few months of his death.

Strict integrity and private worth, with correſponding purity of morals, gave the higheſt value to a character, which, from other qualities and attainments, was much reſpected and eſteemed. Upon all occaſions, even in the gayeſt hours of ſocial intercourſe, the Doctor maintained a conſtant attention to propriety. He had ſerious and juſt impreſſions of religion, which appear occaſionally in his papers, and may be traced even in the midſt of ſome of his mathematical inveſtigations;[*] but he was uniformly reſerved in expreſſing particular opinions on the ſubject of religion; and from his ſentiments of decorum, he never introduced religion as a ſubject of converſation in mixed ſociety, and all attempts to do ſo in his clubs were checked with gravity and deciſion.

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[*] As an example, the following date of the ſolution of a problem, which happened to be his birth-day, may be quoted, as it ſtands in one of his MS. volumes:

"14 Octobris 1764
"14 Octobris 1687
"77
"DEO Optimo Maximo, Benigniſſimo Servatori, ſit laus et gloria."

Dr. SIMSON in his perſon was tall and erect, and his countenance, which was handſome, conveyed a pleaſing expreſſion of the ſuperior character of his mind.* His manner had always ſomewhat of the faſhion which prevailed in the early part of his life, but was uncommonly graceful. He was ſeriouſly indiſpoſed only for a few weeks before his death, and through a very long life had enjoyed an uninterrupted ſtate of good health. He died on the firſt of October 1768, when his eighty-firſt year was almoſt compleated; having bequeathed his ſmall paternal eſtate in Ayrſhire to the eldeſt ſon of his next brother, in whoſe family it ſtill remains.†

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* In the College hall of Glaſgow is a portrait of Dr. SIMSON, which, though painted when he was in the vigour of life, yet expreſſes a likeneſs to his countenance and figure as they appeared in advanced age. On the title-page of this Memoir is placed a ſmall engraving from this picture, with an inſcription written by the late Dr. MOOR, Greek Profeſſor at Glaſgow, happily expreſſing Dr. SIMSON's diſapprobation of the modern uſe of algebra in geometry, and his ſingular merit and ſucceſs in reſtoring the ancient analyſis. See the Edinburgh Tranſactions, vol. i. p. 75. Hiſtory.

† Dr. SIMSON, from his conſtant reſidence in the College, and his engagement in ſtudy, gave little attention to the improvement of his eſtate, which deſcended to him encumbered with debts, and which became much more productive to his heir than it had been to himſelf. He was at the ſame time well acquainted with buſineſs, having been for a great number of years Clerk to the Faculty, and in that character he had the chief management of the property and other affairs of the College, which he conducted with his habitual regularity and accuracy. In his manner of living he was ſimple and unexpenſive, except in collecting a valuable mathematical library, which he bequeathed to his College. But from an income, ample with reſpect to his habits, he made no accumulation; having enjoyed the ſatisfaction of advancing young relations, in his own time, by many acts of generoſity.

NOTES.

NOTES.

NOTE A. p. 28.

FROM APOLLONIUS and his commentators to MYDORGIUS and others of the ſeventeenth century, Dr. SIMSON conſiders this branch of geometry as ſtationary for that long period. It has been properly obſerved however by different authors, that the points, in ſubſequent times called *Foci,* were aſcertained by APPOLLONIUS only in the ellipſe and hyberbola, but not in the parabola. The Rev. Dr. ROBERTSON, Savilian Profeſſor of Aſtronomy at Oxford, in his learned Hiſtory of Conic Sections, annexed to his Treatiſe on that ſubject,* after mentioning this obſervation, adds, that in two ſmall tracts, now very rare, edited by GOGAVA, Louv. 1548, the firſt mention is made, ſo far as he knew, of the focus of the parabola.† GOGAVA conſiders the ſecond tract (*De Speculo Ustorio*) as of Arabic origin, and very ancient; and alſo that VITELLO had borrowed from it. VITELLO flouriſhed about 1470, near a century before GOGAVA; and in his *Optics*, prop. 42, 43, book ix. (page 400, edit. Reſneri,) the focus of the parabola, and the application of it in forming a concave parabolic mirror are diſtinctly deſcribed, but without ſtating the ſource from which he derived them.

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* *Sectionum Conicarum, Libri VII.* &c. Oxon. 1792, pp. 340, 362.

† This publication appears to be a firſt edition, though other dates are given to it by writers who quote it; and it has a Preface by GEMMA FRISIUS. The ſecond of theſe two ſmall tracts ſeems to have been generally known before the edition by GOGAVA. ROGER BACON quotes it; and VERNERUS, in his tract *De Cubo Duplic.* Nurem. 1522, append. 12ma. plainly alludes to it. MAUROLYCUS alſo, in a miſcellaneous volume, (Meſſanæ, 1558,) ſtates

It is plain, however, that the point, afterwards called the focus of the parabola, was uſed by PAPPUS, though not formally diſtinguiſhed or named, in prop. 238, lib. vii. (fol. 303. b. edit. 1588,) which is a *Locus* to the parabola. But he does not mention the property, eaſily flowing from that which he aſſumes, on which is founded the conſtruction of the parabolic reflecting mirror; and it is not improbable that this laſt property had not been obſerved in the time of PAPPUS, and that it was diſcovered long after by the Arabian opticians. This Prop. of PAPPUS muſt have been well known to Dr. SIMSON, though he appears not to have conſidered it as requiring notice in the ſhort hiſtory of Conics in his preface.

It may be convenient in this place to make an obſervation on another portion of the preface to Dr. SIMSON's *Conics*, viz. the paragraph beginning, "*Propositionum jamdudum, &c.*" (p. vi. Præfat. prim. et ſec. edit.) and ending "*contulerint.*"

This paragraph was added in the firſt edition, (1735,) in conſequence of an objection made by Mr. MAC LAURIN to a Propoſition of the Doctor's, as not containing a ſufficient acknowledgment of a communication by Mr. M. on the ſubject of it. Dr. SIMSON, in a letter to a common friend not named, which remains among his papers, ſtates this circumſtance. He ſays, "I "have not time to write about the Propoſition Mr. MAC LAURIN objects againſt. "To take away all controverſy in this matter, I ſend you an addition to the "preface;" viz. the above-mentioned paragraph. After ſome farther explanation of his anxiety to do juſtice to Mr. MAC LAURIN he ſays, "you may "ſhew him this addition to the preface, before printing it."—The explanation

the nature of this tract, obſerving, that it had been erroneouſly attributed to ARCHIMEDES, and it is in an abſtract of the works of ARCHIMEDES that he mentions it. He conſiders it, however, to be the work of PTOLEMY, without giving any authority. See the above mentioned volume of MAUROLYCUS, fol. 72. It was printed indeed ſome years after GOGAVA's publication; but probably MAUROLYCUS had not ſeen theſe ſmall tracts, as he makes no mention of them, or of the editor GOGAVA. The firſt tract, *De Parabola*, is ſuppoſed to be of the time of ROGER BACON, about 1270; but the ſecond, *De Speculo Uſtorio*, is much more ancient, being quoted by ROGER BACON.

ſeems to have been quite ſatisfactory, for in the following year, 1736, the mathematical correſpondence between Dr. SIMSON and Mr. MAC LAURIN continued on its uſual friendly footing.

Among Dr. SIMSON's papers there is an account of his *Conics*, drawn up ſoon after the publication, in the form of a letter, and in the name of ſome friend of the Doctor's, apparently with the deſign of being tranſmitted to a diſtance, and perhaps for publication.* It is a ſort of enlargement of the preface, giving a more particular detail of the author's views in compoſing the treatiſe, and of the improvements which he had made in it. The following paſſage in it gives a more particular account of the communication to Mr. MAC LAURIN. “The Scholium in p. 198, (firſt edit.) and the Prop. it flows from, contain a very general and uſeful property of the Sections, which the author diſcovered upon this occaſion. When Mr. MAC LAURIN, the learned profeſſor of mathematics in the univerſity of Edinburgh, was going to France in the year 1723, Mr. SIMSON communicated to him a *Locus* of PAPPUS Alexand. which he had reſtored, viz. *Si a tribus punctis datis in recta linea ducantur tres rectæ lineæ, et ipsarum intersectiones duæ tangant rectas positione datas, tanget reliqua intersectio rectam lineam positione datam:* which was afterwards printed in the Philoſophical Tranſactions. When Mr. MAC LAURIN had come home from France, he told Mr. SIMSON that he found, when the three given points were not in a ſtraight line, all the reſt remaining as in the former caſe, that the *Locus* deſcribed would be a conic ſection, and that by help of this *Locus* he could eaſily deſcribe a conic ſection through five given points: This *Locus*, and method of deſcription, he gave Mr. SIMSON without any demonſtration, which he ſaid he had made by help of a *Lemma* in Sir ISAAC NEWTON's *Principles*; and it was in ſearching for the demonſtration of them that Mr. SIMSON found out the Propoſitions in the Scholium, and thoſe they flow from, which he did in the ſame method

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* Since writing this account, Mr. FRYER of Briſtol, who is engaged in a hiſtory of mathematics, pointed out to me a letter from the celebrated Profeſſor HUTCHESON of Glaſgow, on Dr. SIMSON's *Conic Sections*, which he had found in the *Bibliotheque Raisonée* for April, May, and June 1735; and which, on compariſon, I aſcertained to be the very letter here alluded to. In that journal it is tranſlated into French, and Profeſſor HUTCHESON is mentioned as an old friend of M. SMITH, one of the publiſhers, at Amſterdam. The letter is dated Glaſgow, 28 Feb. $173\frac{4}{5}$.

"they are now printed in his book." The copy of this letter, found among Dr. SIMSON's papers, is without any ſignature, but it is in the Doctor's handwriting.*

In the note * page 29, the proper reference is to BOSCOVICH *Sect. Con. Elementa.* art. 275.

NOTE B. p. 43.

To the examples mentioned by Dr. SIMSON, of miſapprehenſions entertained by eminent mathematicians reſpecting Poriſms, may be added that of CASTILLON. In his commentary on the *Arithmetica Univerſalis,* (Amſt. 1761,) p. 216, vol. 1, after quoting a paſſage from MARINUS's preface to the Data, he adds, "Hæc obſervatio mihi viam aperuit ad reſtituenda EUCLIDIS Poriſmata;" and afterwards, page 268, when ſpeaking of the ancient problem of the Tactions, he adds, "omnia ſoluta habeo, et me editurum ſpero propediem cum Porismatis "EUCLIDIS reſtitutis.' The ſame author, in his commentary on the appendix, vol. ii. page 264, ſhews more particularly his miſapprehenſion of the nature of Poriſms, by ſuppoſing them to be the conſtructions of EUCLID's data; and then he adds, "ad hunc (ſc. librum Poriſmatum) non pauca colligere "cæperam, ſed cum audiviſſem virum doctiſſimum ROBERTUM SIMSONUM, "rem perfeciſſe, et ſua ſcripta reliquiſſe, ab incepto deſtiti, ſperans et rogans ut "tanti viri cogitationes in publicam lucem emittantur." Dr. SIMSON had got this book a few years before his death, and was much amuſed with theſe obſervations of the author. It is very ſurpriſing that ſo many reſpectable mathematicians deceived themſelves on the ſubject of Poriſms, eſpecially thoſe who were apprized of the failure of Dr. HALLEY, who candidly acknowledges it, but who, of all who attempted the inveſtigation of them before Dr. SIMSON, ſeemed to be, from his talents and his knowledge, the moſt likely to ſucceed.

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* In this letter alſo is given a ſolution of the problem, "To deſcribe a Conic Section "which ſhall paſs through two given points, and touch three ſtraight lines given in poſition."—This he eaſily deduces from Prop. 12. b. v. (1ſt ed.) by means of the before-mentioned *Locus,* publiſhed in the Phil. Tranſactions for 1723, p. 330, without the transformations employed in Prop. 25 and 26 of NEWTON's *Princip.* lib. 1.

Note C. p. 22.

“ I ſubjoin the remainder of Dr. **Simson**’s letter to Dr. **Jurin**, of which the firſt part is in the text at page 22. “ At his (Dr. **Halley**’s) deſire I have
“ conſidered that paſſage as narrowly as I could, and ſhall be very glad to have
“ his opinion concerning the following conjectures; ſince, both with reſpect
“ to the language and the matter, I eſteem him the moſt ſkilful judge.

“ The firſt thing to be conſidered is, that the figures **Pappus** ſpeaks of in
“ this Propoſition (for I ſuppoſe σχήματος, or ſome ſuch word, ought to be under-
“ ſtood to be joined with ὑπὸ &c.) are ſuch as are made by four ſtraight lines,
“ either all of them interſecting one another, or two of them, at the moſt,
“ parallels: (the caſe of the parallelogram, becauſe of its plainneſs, being
“ omitted.) I mean, **Pappus** ſpeaks of the complex figures made by the
“ actual interſections of all the lines, and not of ſimple Trapezia, whoſe ſides,
“ if produced, would meet: this is plain from his ſuppoſing three points of
“ interſection in their ſides; ‘ Si dentur, in earum una, puncta tria.’

“ Of the three kinds of theſe figures he mentions, that which he calls
“ παραλλήλον is eaſily, and I think certainly, determined, from ſome places in
“ his Lemmas, to be the figure made by two parallel lines meeting two others
“ which are not; ſo that either of theſe two figures A or B is the παραλλήλον.*

“ The places which verify this are firſt in Prop. cxxxvi. at the letter C in
“ page 247† (of the edition at Ven. 1589,) **Commandine** has it, ‘ ita KH
“ ad HL hoc eſt in lineis parallelis GH ad HM.’ where the Greek has
“ τȣτ’ ἔστιν ἐν παραλλήλω &c. and the figure GHMLK‡ is plainly ſuch as has been
“ deſcribed above. Again, in Prop. cxxxv. fol. 246, there occurs, “ ut FM
“ ad DH, ita in lineis parallelis FK ad KH;’ which I doubt not will be in
“ the Greek MSS. ἐν παραλλήλω, and the figure is the ſame as in the laſt.

“ The ſame way of ſpeaking is twice uſed in the firſt demonſtration of the
“ firſt Lemma, fol. 238.§. And all the figures to which theſe two paſſages refer
“ are of the other kind, (viz. fig. B.) except one. And from this it appears
“ that παραλλήλον is promiſcuouſly applied to both.

* Fig. A and B. † This ſhould be fol. 246. b. ‡ Fig. 1.

§. It appears that the MS. uſed by **Commandine** had ἐν παραλλήλω in this prop. (127.) See his Note D.

"It is alſo plain, that the paſſage in fol. 242, at letter B, is not corrupted, "as COMMANDINE ſuſpects; it is καὶ γὰρ ἡ θκ πρὸς τὴν θε ἐν ϖαραλλήλω, which he "ſhould have rendered into Latin thus, 'ut autem AF ad FG, ita LH ad 'HM, etenim in eadem ratione (ſc. cum utriſque illis rationibus) eſt HK ad 'HG ἐν παραλλήλω, viz. eſt AF ad FG ut HK ad HG εν' ϖαραλλήλω AFGHK," 'and LH ad HM ut HK ad HG, ἐν ϖαραλλήλω LHKGM.'

"Having determined the figure which PAPPUS calls ϖαραλλήλον, the reſt, viz. "thoſe where none of the lines are parallel, muſt belong to other two kinds, "ὑπτίον and παρυπτίον. The firſt of which words uſually ſignifies *supinus, re- "supinus, restrorsum vergens*, and is applied by ARISTOTLE to the recurved "horns of Oxen. Now all the figures whatſoever that can be made by the "interſecting of four ſtraight lines none of which are parallels, are with "reſpect to their form, of one kind; and have every one a reverſed or retroflexed "angle, which will agree very well with the ſignification of ὑπτίος *restrorsum* "*vergens*. But, then, if every one of theſe figures be ὑπτίον, what comes of "the remaining branch of the diviſion παρυπτίον? This is a difficulty cannot be "taken away, if we ſuppoſe PAPPUS to ſpeak of the form of theſe figures "abſolutely conſidered; but may be removed, if it be found he has regard "to the twofold manner in which they are formed, by two lines drawn "interſecting two other lines, and ſo as to give a juſt reaſon of uſing theſe "terms ὑπτίον and παρυπτίον reſpectively.

"I ſhall firſt explain the two ways after which theſe figures are formed, ſo "as to give riſe to the names PAPPUS uſes, and then ſhew what ground there "is to think he conſidered them thus, and not abſolutely with reſpect to their "form.

"FIG. 2. Let two lines AB, AC, be drawn, interſecting two others DB, DC; "then if the angles BAC, BDC, made by theſe lines, lie one over another, *i. e.* "ὑϖτιῶς; or which is the ſame thing, if one of them, BDC, be the reverſed "angle of the figure; then ſuch lines may be ſaid to make the ὑϖτίον σχῆμα.

"FIG. 3. But when the two lines AB, AC, are drawn interſecting two "others DB, DE, ſo that the two angles BAC, BDE, made by theſe lines, "do not lie below one another, but one of them is upon one ſide and the "other upon the other hand of the ὑπτία γωνία AFD, which they form by their "two neareſt ſides AC, DE, and ſo lie beſide (παρὰ juxta) the reverſed angle, "as alſo at one another's ſides; then theſe pairs of lines may be ſaid, with "reſpect to the angles they make, to form the παρυπτίον σχῆμα.

“ And that Pappus had regard to theſe two different ways in which one pair “ of lines meet another pair, I think will appear plain enough by comparing “ Prop. cxxxvi. with Prop. cxlii.; in each of which he conſiders the figures we “ are now ſpeaking of, as formed by two lines interſecting two others; and theſe “ figures are not only, abſolutely conſidered, exactly the ſame, but the things “ demonſtrated are alſo the ſame, viz. the converſe of Prop. cxxix.; nor is there “ any thing to give occaſion to the ſplitting it into two caſes, other than the “ conſideration of the different ways the one pair of lines meets the other pair; “ viz. in Prop. cxxxvi. the lines DH, HE, meet the lines BAE, DAG, ὑπτίως, “ or ſo as to form the ὕπτιον σχῆμα. And in Prop. cxlii. the lines BD, DE, meet “ the lines AB, AC, παρυπτίως, or ſo as to form the παρύπτιον σχῆμα with reſpect “ to their angles.

“ Some, perhaps, may incline rather to give παρὰ the negative or contrary “ force, when joined with ὕπτιος; becauſe the angles made by the pairs of “ lines which form the figure, are the internal oppoſites of two triangles, of “ which the reverſed angle is the common external. I ſhall leave this to others “ to determine; and ſhall only ſay, I do not remember παρὰ is uſed in this ſenſe “ in any mathematical term; for in Euclid’s Data I think it is quite amiſs “ to tranſlate it *contra*.”

He adds a Poriſm without demonſtration, which it is unneceſſary to inſert, and concludes, with an apology for writing in haſte,

“ I am, Sir, your much obliged and
“ affectionate Servant,

“ Glasgow, Jan. 10, 172¾. ROBERT SIMSON.”

Note [2 C.] p. 49.

On the ſubject of Poriſms, Dr. Simson was certainly reſerved; and this probably in part aroſe from a natural deſire of publiſhing his diſcovery of them in a compleat form, without the riſk of his being anticipated, which from partial communications was not unreaſonably to be apprehended. Candour was a ſtriking point of his own character; and we may obſerve in his works the preciſion with which he remarks the inventions, the improvements, and even the ſmalleſt hints contributed by his friends;* and perhaps he felt ſome diſſatisfaction from not having always met with the ſame liberality. Some inſtances were well known, and there were reports of others; but though they may account for his reſerve, it is neither of importance to his reputation, nor in any reſpect expedient, to inveſtigate or to record them. His reſerve on the ſubject of Poriſms I had frequent opportunities of remarking, though favoured with much familiar converſation with him on mathematical ſubjects. From PAPPUS, and other writers, by whom Poriſms were mentioned, and alſo from ſome very diſtant alluſions from Dr. SIMSON, I certainly had got a general, but as I afterwards learnt, an imperfect notion of Poriſms. I conceived them to be entirely geometrical, and that they were a claſs of Propoſitions in which certain points or lines were to be found, which might have a *general* property expreſſed in the enunciation. I occaſionally ſubmitted to Dr. SIMSON ſome Propoſitions which I conſidered to be of that claſs; but without admitting or denying them to be Poriſms, with ſome pleaſantry he ſaid they were Propoſitions. It appears from his *Poſthumous Works*, p. 459, that, with his uſual accuracy, a ſimple Propoſition of that kind (which I had ſtated to him as a problem) had been entered in the fair MS. of the Poriſms from which the treatiſe was printed.

NOTE D. p. 52.

As every notice of the Poriſms in former times is intereſting, I ſhall inſert ſome extracts from PROCLUS reſpecting them, in his Commentary on the firſt book of EUCLID's *Elements*. I quote from HERVAGIUS's edition, the only printed one, though very erroneous, as is obſerved by BAROCIUS in his tranſlation, who ſays it is "dilaniatum potius quam impreſſum." But as BAROCIUS had acceſs to ſeveral MSS. from which he choſe the beſt readings, his tranſlation, where it differs from the printed Greek, is a better authority; I therefore annex the tranſlation, both it and the original being ſcarce.

The firſt mention of the Poriſms is in page 58 of HERVAGIUS's edition, and in page 121 of BAROCIUS, viz. in the Commentary on the firſt Prop. of the *Elements*, where he profeſſes to give only a brief account of ſeveral mathematical terms, ſuch as Lemma, Poriſm, Caſe, &c.* The notice of the Poriſm is as follows:

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* See SIMSON's *Conics*, 2d edit. præfat. p. vi. and Prop. 19, lib. 5, at the end. See alſo *Loci Plani*, p. 223.

"Τὸ δὲ πόρισμα λεγέται μὲν καὶ ἐπὶ προ-
"βλημάτων τινῶν οἷόν* τὰ ʼΕυκλείδει γεγραμ-
"μένα, πορίσματα λεγέται δε ἰδίως ὅταν ἐκ τῶν
"ἀποδεδειγμένων ἄλλο τὶ συναφανῆ θεώρημα
"μὴ προθεμένων ἡμῶν ὃ καὶ δια τοῦτο πόρισμα
"κεκλήκασι ὥσπερ τι κέρδος ὂν τῆς ἐπιστη-
"μονικῆς ἀποδείξεως πάρεργον."

* ὅια.

"Corollarium vero, dicitur quidem "et de quibuſdam problematibus ut "corollaria, quæ EUCLIDI aſcripta "ſunt. Dicitur autem proprie corollarium, cum ex iis quæ demonſtrata "ſunt quoddam aliud theorema ap"paruerit, nobis minime proponen"tibus, quod etiam propterea corol"larium vocarunt, tanquam lucrum "quoddam, quod ſit præter gignentis "ſcientiam demonſtrationis propo"ſitum."

This very ſhort deſcription is unſatisfactory, but a ſubſequent paſſage (PROCLUS, HERVAGII, p. 80. BAROC. p. 173.) is more particular and intelligible:

"Ἔντι τῶν γεωμετρικῶν ἐστὶν ὀνομάτων το
"πόρισμα. τοῦτο δὲ σημαίνει διττόν. καλοῦσι
"γὰρ πορίσματα καὶ ὅσα θεωρήματα συγκα-
"τασκευάζεται ταῖς ἄλλων ἀποδείξεσιν ὅιον
"ἕρμαια καὶ κέρδη τῶν ζητούντων ὑπάρχοντα,
"καὶ ὅσα ζητεῖται μὲν ἐυρέσεως δε χρήζει
"καὶ ὄυτε γενέσεως μόνης ὄυτε θεωρίας ἁπλῆς.
"ὅτι μὲν γὰρ τῶν ἰσοσκελῶν αἱ πρὸς τῇ βασει
"ἴσαι, θεωρῆσαι δεῖ, καὶ ὄντων δὴ τῶν πραγμάτων
"ἐστιν ἡ τοιαύτη γνῶσις. τὴν δὲ γωνίαν δίχα

"Unum quid geometricorum no"minum corollarium eſt; hoc autem "duplex quidpiam ſignificat; vocant "enim corollaria quæcunque etiam "theoremata, una cum aliorum de"monſtrationibus probantur, veluti "lucra inexpectata atque emolu"menta quærentium exiſtentia: Et "quæcunque quæruntur quidem, in"ventione autem indigent, et neque "generationis ſolæ cauſa quæruntur "neque ſimplicis contemplationis; "nam quod quidem 'Æquicrurium "qui ad baſim ſunt anguli æquales "ſunt,' contemplari oportet, exiſten"tiumque rerum hujuscemodi cog-

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* "Age de iis etiam quæ his annexa ſunt breviter diſſeramus, quid Sumptio (λῆμμα) "quid Caſus, quid Corollarium (πόρισμα), quid Inſtantia, quid Inductio."——BAROC. Verſio, p. 120.

" τεμεῖν ἢ τρίγωνον συστήσασθαι, ἢ ἀφελεῖν ἢ " θέσθαι, ταῦτα πάντα ποίησιν τίνος ἀπαιτεῖ. " τοῦ δοθέντος κύκλου τύ κέντρον εὑρεῖν ἢ δυο " δοθέντων συμμέτρων μεγεθῶν τὸ μέγιστον καὶ " κοινὸν μέτρον εὑρεῖν ἢ ὅσα τοιάδε μεταξύ " πως ἐστι προβλεμάτων καὶ θεωρημάτων. οὔτε " γὰρ γε ἐσεις εἰσὶν ἐν τούτοις τῶν ζητουμένων " ἀλλ' εὑρέσεις, οὔτε θεωρία ψιλὴ. δεῖ γὰρ " ὑπ' ὄψιν ἀγαγεῖν, καὶ πρὸ ὀμμάτων ποιή" σασθαι τό ζητούμενον. τοιαῦτα ἄρα ἐστι καὶ " ὅσα πορίσματα Ἐυκλείδης γέγραφε, βιβλία " προβλημάτων* συντάξας. ἀλλά περὶ μὲν τῶν " τοιούτων πορισμάτων παρείσθω λεγειν. τὰ δὲ " ἐν τῇ στοιχειώσει πορίσματα συναναφαίνεται " μὲν ταῖς ἄλλων ἀποδείξεσιν, αὐτὰ δὲ προη" γουμένης τυγχάνει ζητήσεως. ὅιον καὶ τὸ νύν " προκείμενον." &c.

" nitio eſt. 'Angulum autem biſa" riam ſecare,' vel 'triangulum con" ſtituere,' vel 'rectam lineam æqua" lem abſcindere, vel ponere,' hæc " omnia ut aliquid fiat poſtulant. " 'Dati vero circuli centrum reperire,' " vel 'duabus magnitudinibus com" menſurabilibus datis, maximam ip" ſarum communem menſuram in" venire,' vel quæcunque id genus " alia, quodammodo inter proble" mata atque theoremata ſunt; neque " enim quæſitorum ortus in his, " neque ſola contemplatio ſed inventio " eſt. Opus eſt ſiquidem quæſitum " in conſpectu et præ oculis ponere. " Talia igitur ſunt quæcunque etiam " corollaria EUCLIDES ſcripſit, quippe " qui libros corollariorum conſtruxit. " Verum de hujuſcemodi quidem co" rollariis dicere prætermittatur. Quæ " autem in elementari inſtitutione ſunt " corollaria, ſimul quidem cum alio" rum demonſtrationibus apparent, " ipſa vero non præcipue quæruntur, " veluti id quod in præſentia propo" nitur;" viz. Cor. 15. 1 Elem.

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† From the ſenſe, this word ought to be πορισμάτων, and not προβλημάτων; for whatever be the origin of that claſs of Propoſitions, there can be no doubt of their being always known and diſtinguiſhed by the term πορίσματα. BAROCIUS accordingly tranſlates it in this place by "*corollariorum*;" moſt probably from his having found that reading in a MS. which he relied upon. In the MS. of PROCLUS, in the Bodleian Library, which is eſteemed valuable, προβλημάτων is in the text, but on the margin πορισμάτων is written in a character much reſembling the text. In this ſentence γ probably is omitted after βιβλία.

In this extract is a very explicit ſtatement of the two very different ſpecies of *Poriſms*, (or *Corollaries*, as they are named by **Barocius**,) viz. the Poriſms compoſed by **Euclid**, a curious and difficult claſs of Propoſitions, requiring inveſtigation, as well as conſtruction and demonſtration; and the Poriſms or corollaries of **Euclid**'s Elements, which reſult from the demonſtration of other Propoſitions, and which often preſent themſelves unexpectedly. **Proclus** proceeds to illuſtrate this latter claſs of Poriſms by the example of a corollary annexed to 15 Prop. 1 Elem. the preceding extract being part of his commentary on that Propoſition. He then gives a more diffuſe deſcription of this corollary of the elements, of which a portion is annexed.

(Hervagii. p. 80. Baroc. p. 174.)

"Ἔστιν οὖν τὸ πόρισμα θεώρημα διὰ τοῦ ἄλλου
"προβλήματος ἢ θεωρήματος ἀποδείξεως,
"ἀπραγματεύτως ἀναφαινόμενον. οἷον γὰρ
"κατὰ τύχην περιπίπτειν ἐοίκαμεν τοῖς πο-
"ρίσμασιν, οὐ γὰρ προθεμένοις, οὐδὲ ζητήσασιν
"ἀπαντᾷ, ὅθεν αὐτὰ καὶ τοῖς, ἑρμάιας
"εἰκάσαμεν, καὶ ἴσως οἱ δεινοὶ τὰ μαθημα-
"τικὰ, κατὰ ταύτην αὐτοῖς ἔθεντο τὴν ἐπωνυμίαν,
"ἐνδεικνύμενοι τοῖς πολλοῖς καὶ ἐπὶ τὸ φαι-
"νόμενον κέρδος ἐπτοημένοις* ὅτι ἄρα τὰ
"ἀληθῆ θεοῦ δῶρα καὶ τὰ ἕρμαια ταῦτα ἐστίν
"οὐχ οἷα ἐκείνοις δοκεῖ."

* Forſan ἡδομένοις.

"Corollarium eſt theorema quod "ex alius problematis vel theorematis "demonſtratione ex improviſo emergit: nam veluti caſu quodam in "corollaria incidere videmur, nec "proponentibus enim nobis, neque "etiam quærentibus obviam ſeſe "offerunt. Unde hæc quoque lucris "aſſimilavimus: et fortaſſe mathematicarum rerum periti hoc ipſis "impoſuere nomen, oſtendentes vulgo, quippe quod apparenti gaudet "lucro, quod utique vera **Dei** munera, veraque lucra hæc ſunt, non "autem quæ illi videntur," &c.

Then follow ſeveral diſtinctions of corollaries, into arithmetical and geometrical, into thoſe ariſing from theorems and thoſe from problems, with ſome others of no particular importance. Theſe extracts, it muſt be allowed, are not expreſſed with uniform clearneſs, but they diſcriminate ſufficiently the Poriſms of **Euclid** from the corollaries of the Elements; and they correſpond with the more general expreſſions of **Pappus** on this ſubject, which **Proclus** in theſe paſſages plainly had in view. For illuſtrating the

the diſtinction, he premiſes that the 5th Prop. 1 Elem. is a theorem, and requires demonſtration: alſo, that the 1ſt, 3d, and 9th Propoſitions are problems, each requiring ſomething to be done or conſtructed; he then adds, that to find the centre of a given circle, (1. 3. Elem.) or to find the greateſt common meaſure of two magnitudes, (2. 7. Elem.) or ſuch like things, are in ſome ſort of an intermediate character between problems and theorems. For the conſtruction of theſe things ſought is not given in the enunciation, but invention is requiſite for finding that conſtruction, and alſo for diſcovering the demonſtration; for it is neceſſary to exhibit to the eye the conſtruction of the things ſought. Such, ſays he, is the nature of the ſecond kind of Poriſms, of which Euclid compoſed books.* Of theſe obſervations, not altogether clear, I have given, in a ſort of paraphraſe, the meaning as I underſtand it to be. It is plain he was acquainted with the obſervation of Pappus, that Poriſms are of a middle nature between problems and theorems, though ſome doubt may be entertained, whether the examples which he quotes be properly ſtated as Poriſms, as was obſerved in a former note p. 47. It may be remarked here, that Proclus, in referring to the 2d. 7th Elem. as a Poriſm, ſuppoſes that there may be arithmetical Poriſms as well as geometrical, and this Propoſition is altogether unconnected with *Loci*, or even with geometrical poſition of any kind.

I may obſerve further in this place, that *Loci*, which by Pappus are reckoned a claſs of Poriſms, have alſo ſomewhat of the intermediate character between problems and theorems; though they are generally reckoned to belong to the latter claſs. In the *Locus* the conſtruction muſt be inveſtigated, as in the *Poriſm*; and every *Locus* is eaſily convertible into a Poriſm.

The etymology of the Poriſm, or corollary of the Elements, from ϖόρισμα ſignifying gain, may be right; and the other meaning of the Greek term ϖόρισμα, implying inveſtigation, may be the ground of its being applied to the Poriſms of Euclid. In the firſt ſenſe it is the common corollary, which is an acquiſition (or gain) from another Propoſition, from the demonſtration of which it reſults often unexpectedly.† Πόρισμα alſo, from the other ſignification of the word, properly denotes any thing to be inveſtigated, which correſponds with the character of the Poriſms of **Euclid**.* And thus, without any connection between theſe two claſſes of Propoſitions, they may incidentally, from the two unconnected meanings of a Greek word, have obtained the ſame name.

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* See Note, p. 90.

† Corollarium, aſſumed by Barocius as the proper tranſlation of ϖόρισμα, may expreſs the firſt meaning of it, but has no connection with the other.

It is neceſſary, however, to remark, that the firſt extract from **Proclus**, (p. 58, **Hervag.**) containing the very ſhort deſcription of Poriſms, does not, as it ſtands, accurately correſpond with the detailed and more intelligible deſcription in the ſecond. If the word ϖροβλήματων in the former paſſage be the true reading, it might thence certainly be urged, that Poriſms have a reference to problems; and an argument might be derived in favour of the opinion, that the ancient notion of Poriſms was ſimilar to that propoſed by Mr. **Playfair**. But this preliminary deſcription of Poriſms by **Proclus** is ſurely too ſhort and general to be the foundation of any ſuch inference, without other more preciſe authorities; and the ſubſequent paſſage, which is much more particular and intelligible, contains no ſuch reference to problems. Beſides, it is well known, that the only printed edition of **Proclus** is very erroneous.† If it may be ſuppoſed that ϖροβλήματων is printed for ϖορίσματων,‡ as it moſt certainly is in another place in the ſecond extract, the two paſſages will be more conſiſtent; and though the expreſſion in the firſt be ſhort and inexplicit, it being only an introductory notice of Poriſms, yet it may be underſtood. I conceive, therefore, that in both extracts his object is the ſame, viz. to diſtinguiſh the Poriſms compoſed by **Euclid** from the Poriſm which is the common corollary; and he does it on the ſame principle, though more briefly in the firſt than in the ſecond: and if *ϖορίσματων* be aſſumed as the true reading in the firſt paſſage, it may be freely tranſlated thus: "The term Poriſm is applied to "that claſs of Propoſitions of which are the Poriſms compoſed by **Euclid**; "but more properly the term Poriſm is applied to Propoſitions not propoſed by "us, but ariſing from the demonſtration of other Propoſitions; and being thus "an unexpected acquiſition- they obtained that name."

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* Notwithſtanding theſe notices of **Euclid**'s Poriſms, **Proclus** does not ſpeak of their curioſity and importance in the manner that **Pappus** does. And it is ſomewhat remarkable, that when **Proclus** gives a catalogue of the works of **Euclid**, (p. 20, **Hervag.** edit.) he makes no mention whatever of his Poriſms. Dr. **Simson**, moſt probably, had conſidered theſe paſſages in his firſt enquiries on this ſubject; but as they could not give the ſmalleſt aid towards his diſcovery of the true nature of Poriſms, he does not ſeem afterwards to have attended to the remarks of **Proclus**, as there is no alluſion to them among his papers.

† **Barocius**, in the dedication of his tranſlation of **Proclus**, ſays, "quamvis neſcius non "eſſem quod impreſſi fuerant **Basiliæ** quatuor **Procli Diadochi** libri commentariorum in "primum Elementorum **Euclidis**; quos adeo laceros et corruptos vidi, ut nihil boni ex iis "elicere potuerim: editi namque erant perinde ac ſi editi nunquam fuiſſent." Afterwards, having mentioned the different MSS. to which he had acceſs, he adds, "ubi ex iis omnibus "exemplaribus quoad fieri potuit unum integrum feci quod poſtremo e Græca lingua in "Latinam converti, &c."——**Proclus Barocii**, Patav. 1569.

‡ I muſt obſerve, that in the tranſlation by **Barocius**, which, though it includes many corrections of the printed Greek, yet contains many errors, the MS. which he relied on muſt have had ϖροβλήματων in this place, as in the edition by **Hervagius**.

It may be further obſerved, that in the Commentary of **Proclus** on the 1ſt, 1. Elem. there are various diſtinctions of problems ſtated; one of which, viz. the *ϖλεονάζον* and the *ἐλλεῖϖες* may here be properly mentioned. By the former term (exceeding) is meant any problem which has more conditions or data than are neceſſary to the ſolution. As for inſtance, if it be propoſed to deſcribe an equilateral triangle, of which each of the angles ſhall be two thirds of a right angle. This laſt condition is implied in the former, and is therefore ſuperfluous. And if an unneceſſary condition be added, the probability is that it will be inconſiſtent with the others, and render the problem ſo ſtated impoſſible. The other term (deficient) expreſſes the want of a neceſſary condition, by which the problem becomes, according to modern language, indeterminate.* *Ἐλλείϖες δὲ* ἔστι πρόβλημα, το προσθήκης ἄλλης δεόμενον ἵνα ἐκ τῆς ἀοριστίας, εἰς τάξιν καὶ ὅρον ἐπιστημονικὸν ἀχθῇ.† **Hervag.** edit. p. 61.

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* In **Pappus** and **Proclus** the term *ἀδιόριϛος*, commonly tranſlated *indeterminatum*, or indeterminate, is applied to problems which do not require or admit of any determination, a meaning altogether different from the modern uſe of the word: *ἀόριϛος* is indeed uſed by **Proclus** in the modern ſenſe of indeterminate. The following ſentence may be quoted, **Hervagius**, p. 61, and **Baroc.** 127, at the cloſe of the commentary on the firſt propoſition of the Elements: "*φανερὸν οὖν ἐκ τȣ́των ὅτι τὰ κυρίως λεγόμενα ϖροβλήματα, βȣ́λεται* "*τὴν ἀοριϛίαν διαφεύγειν, καὶ μη εἶναι των, ἀϖειραχῶς γιγνομένον. λεγεται δε ὅλως κακεῖνα ϖρο*-"*βληματα δὶα την ὁμωνομίαν τȣ ϖροβληματος.*" &c. "Ex his itaque manifeſtum eſt, quod ea "quæ proprie problemata appellantur, indeterminationem effugere debent, et non eſſe ex "eorum numero quæ infinitis modis fiunt. Problemata tamen et illa dicuntur per problematis "æquivocationem, &c." **Baroc.**

In this paſſage it is intimated that indeterminate Propoſitions are alſo called *problems*, from a double meaning of the word; and among ſuch *problems Loci* muſt of courſe have a place, and therefore *Poriſms* alſo; but it is not eaſy to reconcile the different obſervations of **Proclus** on this ſubject.

If any ſuch notion had exiſted in the times of **Pappus** or **Proclus**, as the ingenious definition of Mr. **Playfair** involves, we might have expected ſome reference or alluſion to it in a diſcuſſion like this concerning problems. **Proclus**, in the preſent ſtate of the text, even with all the ſkill of **Barocius**, is not always intelligible, nor conſiſtent in his different obſervations on the ſame ſubject. See his comment on the 1ſt Prop. **Eucl.**; alſo on the 22d Prop. compared with his comment on the corollary to the 15th Prop.

Note E. p. 57.

The 117th Prop. of the ſeventh book of **Pappus** has become remarkable, from the conſideration which has been given to it by many diſtinguiſhed mathematicians of modern times, more than from any particular excellence belonging to it, as it ſtands in that author. In 1742, an extenſion of it was propoſed by M. **Cramer** of Geneva to M. **De Castillon**, viz. by ſuppoſing the three points not to be in a ſtraight line, as is aſſumed in the Propoſition of **Pappus**; and then the problem was thus expreſſed, "a circle and three "points being given in poſition, to inſcribe a triangle in the circle of which "the three ſides ſhall reſpectively paſs through the three given points." It was not however till 1776, that **Castillon** publiſhed a ſolution, in the Berlin Memoirs of that year. The ſolution is geometrical, and diſtinguiſhed into many caſes. In the ſame volume is an algebraical ſolution by **La Grange**, by means of ſome trigonometrical formulas. In the *Petersburgh Acts* for 1780, are ſolutions of the ſame problem by Meſſrs. **Euler**, **Lexel**, and **Fuss**. In the *Memorie di Mathematica e Fisica della Societa Italiana*, tom. iv. Verona, 1788, are two memoirs reſpecting this problem.* The firſt is by **Gior-**

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† **Hugo de Omerique**, in the fourth book of his *Analyſis Geometrica*, ſeems to have had ſome of the diſtinctions of **Proclus** in view. The *Deficiens Problema* of the latter is called *diminutum* by the former, who gives ſome examples of ſuch problems, in which the ſolution turns out to be a *Locus*.

* The following ſolutions and extenſions of this problem were pointed out to me by Mr. **Playfair**.

DANO DI OTTAJANO, in which is a ſketch of the hiſtory of the problem; and though his ſolutions are geometrical, yet he appears to have had but an imperfect notion of the ancient geometrical analyſis. He reſolves the problem before mentioned, adding a variation of it, viz. that the three lines from the angular points of the inſcribed triangle to the three given points may make angles with the ſides of the triangle equal to given angles; and concludes with an important extenſion to the caſe of a polygon, of any number of ſides, which he inſcribes in a given circle, ſo that the ſides reſpectively ſhall paſs through the ſame number of given points. The other memoir in the ſame volume is by Signor GIAN. FRANCESCO MALFATTI, and contains a good ſolution of the very general problem laſt mentioned; from reducing the caſe of polygons to that of the triangle, by the uſe of two very eaſy theorems. I ſhould have added to the account of the firſt memoir, that the author acknowledges his inability to reſolve the problem algebraically, and expreſſes a wiſh that ſome mathematician would attempt a ſolution purely analytical. Hence alſo this problem may be conſidered as an example of what Dr. SIMSON obſerves, that many geometrical problems are reſolved more eaſily by the ancient analyſis than by the modern.

In the Berlin Memoirs for 1796, is a memoir by LHUILIER, containing an algebraical ſolution of the moſt general caſe of the polygon. He premiſes two trigonometrical theorems, one of which is the baſis of La GRANGE's ſolution of the caſe of the triangle, and by means of them proceeds ſucceſſively from the ſimple to the more complex caſes of polygons; and in all the caſes deduces a quadratic equation, from the ſolution of which the problem is reſolved. The expreſſions of courſe become very complex; and a geometrical conſtruction derived from them would be not leſs ſo. LHUILIER extends the problem alſo to Conic Sections, and adds a ſimilar one reſpecting the ſphere.

I ſhall ſubjoin Dr. SIMSON's ſolution of the caſe, when the three points are not in a ſtraight line; dated Auguſt 30, 1731, many years before this problem acquired celebrity from having employed the ſkill of ſo many eminent mathematicians. The Doctor wrote all his mathematical notes in Latin, and I give this Propoſition in his own words.[*]

[†] In this and ſome other Propoſitions of Dr. SIMSON's copied into theſe notes, the Data are quoted as numbered in the old edition. I may remark alſo, that there are ſome differences in the phraſeology of Dr. SIMSON's early writings from what he uſed in more advanced life, but they are not important.

("Eſt Prop. 117, lib. vii. generalior facta.") Fig. 4.

"Datis tribus punctis A, B, C, et circulo DEF poſitione dato,
"a duobis ex punctis A, B, inflectere ad circumferentiam AE, EB
"occurrentes circulo in D, G, quæ faciant DGC, rectam lineam."

"Factum puta; et ducatur DF parallela ipſi AB junctæ, occurrens circulo
"in F, et juncta FG occurrat AB in H. Quoniam igitur propter parallelas
"eſt angulus BHF æqualis angulo DFG, hoc eſt, propter circulum, angulo
"DEG, erunt triangula AEB, BHG æquiangula, et igitur rectangulum ABH
"æquale erit dato rectangulo (92 Dat.) EBG, et datur AB, quare et BH et
"punctum H, et igitur juncta HC poſitione dabitur; occurrat hæc ipſi DF
"in K, et erit angulus DKH æqualis dato angulo BHK; quare eo deventum
"eſt, ut a duobus punctis datis C, H, inflectantur ad circumferentiam CG GH,
"ita ut DF datum faciat angulum DKH cum ipſa HC; puta factum, et
"ducatur DL parallela ipſi CH, et juncta LF occurrat CH in M, ergo dabitur
"punctum M, ut prius oſtenſum de puncto H; [ſc. quoniam angulus CMF
"æqualis eſt angulo FLD, hoc eſt angulo FGC, igitur æquales ſunt anguli
"HMF HGC, et triangula HMF, HGC, ſimilia, quare et CHM rectangulum
"dato FHG eſt æquale, et datur GH, ergo et MH et punctum M,] et propter
"datum angulum DKH dabitur angulus KDL, et igitur recta LF magnitudine
"dabitur [88 Dat.[quoniam igitur a dato puncto M ad circulum poſitione
"datum ducta eſt MFL, faciens FL quæ circulo intercepitur datam, dabitur
"ML poſitione, et igitur LD et AD, et BE. Q. E. I.

"Compoſitio.

"Fiat rectangulum ABH æquale rectangulo contento ſegmentis cujuſvis
"rectæ quæ a puncto B ad circulum producitur, et jungatur HC, ſiat vero
"rectangutum CHM æquale rectangulo contento ſegmentis cujuſvis rectæ
"quæ a puncto H ad circulum producitur, et a puncto M ducatur MFL quæ
"abſcindat ſegmentum LEDF capiens angulum angulo CHB æqualem, junctaque
"HF occurrat circumferentiæ in G, juncta vero BG eidem occurrat in E, et
"juncta AE eidem occurrat in D. Erunt puncta D, G, C, in recta linea. Jungatur
"DF, et connectentur DL, DG, CG: et quoniam rectangulum ABH æquale
"eſt ex conſtructione, rectangulo EBG, erit angulus BHG æqualis angulo
"E hoc eſt angulo DFG, igitur parallelæ ſunt AB, DF, ergo angulus DKH

" æqualis eſt ipſi BHK, hoc eſt ex conſtructione angulo LDF quare parallelæ " ſunt DL, HK; et propter rectangulum CHM æquale rectangulo FHG in " circulo ſunt puncta C,M,G,F; ergo angulus CGF æqualis eſt [angulo " CMF, hoc eſt] ipſi DLF, et propter circulum anguli DLF et DGF " ſimul æquales ſunt duobus rectis, ergo anguli CGF, DGF, ſimul æquales ſunt " duobus rectis, et igitur recta eſt linea DGC. Q. E. D."—" Tranſcriptum " Aug. 30, 1731."

It may alſo be mentioned that Dr. SIMSON at the ſame time gave two variations of the problem, viz. the ſame things being given to make the line DG parallel to a given ſtraight line; and alſo the ſame things being ſuppoſed, to inflect AEB, ſo that the line DG may make an angle with the line drawn from D to the third given point, equal to a given angle.

NOTE F. p. 58.

The caſe of inclinations reſpecting the Rhombus has always been conſidered as an elegant geometrical Problem, and it attracted Dr. SIMSON's attention in his early ſtudy of PAPPUS. His amendment of the figure of a Lemma in COMMANDINE's PAPPUS, (Prop. 70. lib. vii. PAPPI,) which no doubt was uſed by APOLLONIUS in the ſolution of this problem, and the Doctor's correſponding ſolution of it, are dated Feb. 20, 1723. The following tranſcript was taken from a later copy without date, but is ſubſtantially the ſame with the original paper, with ſome ſlight amendments.

It is proper, however, to mention, that the Problem of the triſection of an arch of a circle is treated of in the iv. b. of PAPPUS; and in Prop. 31ſt it is propoſed to be done by an *inclination* which is a ſolid Problem, there reſolved by an hyperbola. The caſe of *inclinations* employed is that of a rectangular parallelogram, though the ſolution be the ſame in any other parallelogram. Of courſe, a Rhombus being a parallelogram, it is alſo comprehended in the general ſolution. But this caſe of the Rhombus is a Problem in the treatiſe by APOLLONIUS *De Inclinationibus*, and is plane. Dr. SIMSON, however, in his notes on this Prop. of PAPPUS ſhews, that when the Hyperbola is employed in the caſe of the Rhombus, the requiſite circle is deſcribed about the vertex of the Hyperbola, and then the interſections of the Hyperbola

and circle may be determined by plane geometry, and without the deſcription of the Hyperbola. Thus the Problem of the Rhombus, though a caſe of a ſolid Problem, as often happens, becomes plane; and is reſolved in this manner by Dr. SIMSON. But the following ſolution of the Doctor's is better, being no doubt the APOLLONIAN, as the lemma, Prop. 70. vii. PAPPI, is employed.

“ *Prop. de Rhombo cui inservit Prop: 70. lib. vii.* PAPPI.”

[“ Schemate hujus ſc. Prop. 70, emendato ut in Fig. 6.”]

“ Rhombo existente, ABCD, et producta AC ad E, facere EF
“ datam, et quæ ad punctum B vergat. Fig. 5.”

“ Factum puta et producta BC ad G, angulo FCG æqualis fiat BFG, et
“ GE jungatur. Ergo angulus GFE, æqualis eſt ipſi [FCB hoc eſt, ipſi BCA
“ ſeu] GCE. Sunt igitur F, C, G, E, in circulo; quare angulus FGE,
“ æqualis eſt [ipſi FCE vel ei quæ ipſi deinceps eſt, hoc eſt] angulo CDB:
“ et in triangulis FGE, BDC, oſtenſus eſt GFE æqualis angulo DCB
“ æquiangula propterea ſunt triangula. Ergo ut BC ad BD, ita EF ad
“ EG, et data ex hypotheſi eſt FE, quare data eſt EG, ſive ipſi æqualis
“ FG. Eſt autem propter æquiangula triangula BGF, FGC, BG ad GF
“ ut GF ad GC; igitur rectangulum BGC æquale eſt quadrato ex GF:
“ datum proinde eſt BGC rectangulum, et data eſt BG ergo [84 Dat.]
“ dabitur CG, et punctum G propterea datum erit; et data eſt GF magnitudine,
“ et CD poſitione ergo (31 Dat.) poſitione data eſt GF et punctum
“ F, ſed et punctum B, ergo recta BFE poſitione data eſt.

“ Componetur ita: Sit BH data recta, et ut BC ad BD, ita fiat BH ad
“ quartam BK: ad datua vero rectam BC applicetur rectangulum BGC
“ æquale quadrato ex BK, excedens quadrato; factaque GL ipſi BK
“ æquali, centro G intervallo GL deſcribatur circulus qui rectis AC, CD
“ occurrat in E, F, punctis: Erit juncta EF æqualis datæ BH vergetque ad
“ punctum B.

“ Jungantur enim G E et G F; et quoniam GL media eſt proportionalis
“ inter BG, GC, erunt puncta E, F, B, in recta linea per Prop. 70. lib. vii.
“ PAPPI. Et quoniam eſt BG ad GL ſeu GF ut GF ad GC, æquiangula erunt
“ triangula BGF, FGC; quare angulus BFG æqualis eſt ipſi FCG, et angulus

“GFE ipſi [FCB hoc eſt, ipſi BCA ſeu] GCE. Ergo in circulo ſunt puncta F, C, G, E, angulusque FGE æqualis erit [ipſi FCE vel ei qui ipſi deinceps eſt hoc eſt] angulo CDB: æquiangula igitur ſunt CBD, EFG triangula, et ut CB ad BD, hoc eſt ut BH ad BK, ita erit EF ad EG vel FG; æqualis autem eſt BK ipſi FG ſeu GL, ergo æqualis eſt BH ipſi EF. Q. E D.

“Potuiſſet compoſitio æque ac analyſis facta fuiſſe ſine Pappi Lemmate, ſc. jungendo BF et producendo eam ad AC, in E; ita enim non opus fuiſſet Lemmate ad oſtendendum puncta E, F, B, in recta linea eſſe. Hæc autem compoſitio ſtricte loquendo reſpondiſſet problemati quo requiritur a puncto B ducere rectam BFE, et facere EF datam; et non huic quo requiritur producere AC ad E, et facere EF datam quæ ad punctum B vergit; hoc eſt in angulo exteriore Rhombi ECF, ſeu interiore ACF, aptare rectam EF magnitudine datam quæ vergat ad punctum B; et ut compoſitio huic enunciationi ad amuſſim reſpondeat videtur Lemma Pappi utile, et non aliam ob cauſam. R. S.”

“In caſu anguli interioris Rhombi, addatur determinatio ope, Prop. 73, 74, lib. vii. Pappi, et fiant figuræ reſpondentes pro caſu, [Prop. 70. Pappi, et inde in problemate] quando ponenda EF in angulo interiore ACD, et etiam per B punctum; [vel ad B vergens.]

“Lemma in linea ultima, fol. 205, a. Pappi, (Command. 1589) aſſumptum ab eo, eſt ut ſequitur. Vid. fig. Prop. 70. ‘Si a puncto C in diametro DF ducantur ad circumferentiam rectæ CL CK, quæ faciant cum diametro æquales angulos LCF, KCF, erunt ductæ inter ſe æquales;’ cujus demonſtratio facilis eſt. Vide fig. 6. [Schema emendatum, Prop. 70. lib. vii. Pappi.”]

Note G. p. 62.

As Dr. Simson had at one time intended to compoſe a treatiſe on *Loci ad ſuperficiem*, it may be ſatisfactory to ſome readers to ſee his preface, in which he expreſſes this intention; and this, with ſome excerpts from the remaining fragments on this ſubject, may give a general notion of the Doctor's views of this ſubject.

“De *Locis ad ſuperficiem*, quæ ab Euclide aliiſve geometris antiquis tradita fuere, injuria temporis deperdita ſunt, exceptis paucis iis quæ apud

" PAPPUM habentur. Recentiores* autem nihil de *Locis* hiſce cognoſcere
" videntur, exceptis *Locis* quæ ſunt ad ſuperficiem rotatione alicujus ex ſectionibus conicis circa diametros vel alias rectas genitam, cum tamen
" magnus ſit numerus aliorum qui hoc modo minime formari poſſunt. Operæ
" igitur pretium facturum ab eruditis videri ſpero, ſi viam qua *Loci* hi inveſtigari poſſunt exemplis quibuſdam patefaciam, quibus rite intellectis, non
" difficile erit ei qui doctrina *Locorum solidorum* probe inſtructus fuerit,
" *Locos* ad quamvis ſuperficiem, quæ quovis modo ex ſectionibus conicis
" vario ſitu diſpoſitis oriri poteſt, inveſtigare et deſcribere.

" Hi autem omnes [phraſi algebraica] ſequente æquatione continentur, viz.

$$\text{"}\ xx+\frac{a}{b}xy+ax+by+\frac{a}{c}yy+\frac{a}{d}yz+\frac{a}{e}xz+gz+\frac{h}{k}zz=\text{ Dato.}$$

" Signis utcunque mutatis."

The firſt example, as was obſerved in the Memoir, is unfiniſhed. The Doctor had proceeded ſo far as to aſcertain the common ſections of the ſuperficies required with ſeveral planes; ſome others were propoſed, but not inveſtigated, and there is no trace of his having reſumed the inquiry. The portion of this example which is written becomes not a little complex, requiring alſo ſubſidiary Propoſitions; and therefore the enunciation only of the *Locus* is here ſtated; which may be referred to the diagram of the ſubſequent Propoſition. [Fig. 8.]

" Data sint in plano quovis, puta in plano chartæ in quo
" schema est, duo puncta A, B, et juncta BA, in ea sumatur
" quodvis punctum D, et ipsi AD ad rectos angulos, ducatur in
" eodem plano recta DE, et a termino ejus E ad planum erigatur
" ad rectos angulos EF, sitque rectangulum BDA simul cum
" rectangulo DEF, et quadrato quod fit ex EF æquale spatio dato,
" puta (simplicitatis gratia) quadrato ex ipsa AB: quæritur
" superficies in qua omnia puncta F versantur."

To this no equation is annexed.

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* " N. B. ſcripta fuerunt hæc priusquam librum, D. N. *des Courbes a double Courbure*
" uidiſſem, itaque mutanda eſt hæc introductio."

The following is a ſhort example of a *Locus ad ſuperficiem*, found among ſeveral unfiniſhed ſketches of ſuch Propoſitions which were mentioned in the Memoir. It is given as it ſtands in the Doctor's own words.

“ *Locus ad Superficiem.* $xx+\frac{ay^2}{b}+\frac{az^2}{c}=d^2$ [Fig. 8.]

“ Data positione recta linea AB, punctoque in ipsa A, in
“ dato positione plano ABC; si a puncto D in recta AB ipsi
“ ad rectos angulos ducatur recta DE, et a termino ipsius E,
“ erigatur ad planum ABC perpendicularis recta EF; fueritque
“ quadratum ex AD, simul cum spatio quod ad quadratum
“ ex DE, et eo quod ad quadratum ex EF, datas habent rationes,
“ æquale dato spatio, scil. quadrato ex data recta AB. Tanget
“ punctum F, superficiem positione datam.”

“ Sit enim quadratum ex DG, id quod ad quadratum ex DE datam
“ habet rationem, et juncta AG occurrat circulo centro A ſemidiametro AB
“ deſcripto in H, idem vero circulus occurrat DE in K, L, punctis. Quoniam
“ igitur quadrata ex AD, DG, hoc eſt quadratum ex AG, una cum eo quod
“ ad quadratum ex EF datam habet rationem æquale eſt quadrato ex AB, ſeu
“ AH, auferatur commune quadratum ex AG, et reliquum rectangulum KGL
“ æquali erit ſpatio quod ad quadratum ex EF datam habet rationem. Fiat
“ autem ut DG ad DE, ita DK ad DM; quadrata igitur ex hiſce rectis pro-
“ portionalia erunt, et [per. 19. 5.] quoniam quadratum ex DK eſt ad quadratum
“ ex DM ut quadratum ex DG ad quadratum ex DE, erit in eadem ratione
“ data reliquum rectangulum KGL ad exceſſum quadratorum ex DM, DE,
“ hoc eſt, facta DN æquali ipſi DM, ad rectangulum MEN. Datur igitur
“ ratio rectanguli MEN ad rectangulum KGL, hujus autem ratio ad qua-
“ dratum ex EF datur, quare [per 8. Dat.] datur ratio MEN ad quadratum ex
“ EF. Quoniam vero datur ratio DK ad DM, tangit autem punctum K
“ circumferentiam poſitione datam, ſemidiametro, ſcil. AB deſcriptam, tanget
“ punctum M ellipſin poſitione datam, cujus ſemiaxis major eſt ipſa AB;
“ occurrat vero EF ſphæroidi oblongæ hac ellipſi deſcriptæ in O. Et quoniam
“ data oſtenſa fuit ratio rectanguli MEN, hoc eſt quadrati ex EO ad quadratum
“ ex EF, dabitur ratio EO ad EF. Si igitur altitudo EO omnium punctorum
“ O hujus ſphæroidis ſupra planum ABC diminuatur, vel augeatur, in ratione

“ data EO ad EF ſuperficies ſolidi, diminutione hac vel additione producti erit “ Locus punctorum F. Q. E. I.”

Another ſhort Propoſition exhibits a method of deſcribing the Caſſinian curve by means of *Loci ad superficiem.*

PROB. [Fig. 9.]

“ Describere curvam Cassinianam.”

“ Sint A, B, Foci, C centrum, et D junctum in curva; igitur junctis AD “ BD, erit rectangulum ADB æquale dato ſpatio.”

“ Plano in quo eſt curva ducatur ad rectos angulos recta DE ipſi DA “ æqualis, et tanget punctum E conum rectangulum cujus vertex eſt punctum “ A, axis vero plano ABD ad rectos angulos; et quoniam rectus eſt angulus “ EDB, et rectangulum EDB æquale dato ſpatio, tanget idem punctum E “ hyperbolam æquilateram cujus una aſſymptotos eſt DB, altera vero recta “ BF quæ ad planum ADB eſt perpendicularis. Revolvatur igitur hyperbola “ hæc circa BF tanquam axem, et deſcribetur ſolidum hyperbolicum acutum; “ communis vero ſectio ſuperficiei hujus cum ſuperficie coni prædicti, erit linea a “ cujus punctis ſi ducantur perpendiculares ad planum ADB, ipſi occurrent “ in punctis quæ ſunt in curva Caſſiniana, ut patet.”

Another example may be the 28 Prop. lib. iv. PAPPI, as corrected by Dr. SIMSON.

PROB. V. PROP. 28. Lib. iv. PAPPI. [Fig. 7.]

“ Hic igitur lineæ ortus magis mechanicus est, ut dictum “ fuit, geometrice vero per Locos qui ad superficies “ dicuntur”* “ resolvi potest hoc modo.”

“ Sit circuli quadrans poſitione datus ABC, et ducatur, ut contingit, recta “ linea BD, et ad BC perpendicularis EF quæ ad circumferentiam DC proportionem datam habeat. Dico punctum E ad lineam eſſe.

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* Deeſt hoc verbum in MS. BULL.

“ Intelligatur enim a circumferentia ADC recti cylindri ſuperficies, et
“ in ipſa linea ſpiralis [CGH] deſcripta, [incipiens a puncto C, ita ut
“ velocitas puncti deſcribentis in recta CM quæ perpendicularis eſt ad
“ planum ACB, ſit ad velocitatem ipſius CM in quadrante circuli CDA in
“ data ratione quam habet EF ad circumferentiam DC, quæ propterea
“ ſpiralis] poſitione data erit. Sitque HD latus cylindri, et ad planum
“ circuli perpendiculares ducantur EI, BL, et per H ipſi BD parallela
“ ducatur HL, [ipſi EI occurrens in I.] Itaque quoniam proportio rectæ lineæ
“ EF ad circumferentiam DC eadem eſt quæ proportio EI, hoc eſt DH
“ ad eandem DC propter ſpiralem; erit recta EF ipſi EI æqualis. Suntque
“ FE, EI, poſitione, ergo et juncta FI poſitione erit, [N. B. Non dicit FE, EI,
“ poſitione datas, datum non eſt punctum E, ſed intelligit tantum angulum
“ EFB datum eſſe, et EI perpendicularem eſſe ad planum circuli, unde
“ quoniam rectus eſt angulus EFB, erit:] et FI ad BC perpendicularis. Eſt
“ igitur FI in plano [ſc. quod per BC tranſit, et per dimidium anguli recti
“ inclinatum eſt ad planum ſubjectum ABC, angulus enim EFI dimidium
“ eſt recti] quare et punctum I eſt in eodem; atque idem punctum eſt in
“ ſuperficie, fertur enim HL et per lineam ſpiralem CGH, et per rectam
“ lineam BL, et ipſam poſitione datam, ſemper exiſtens [ſc. ipſa HL] parallela
“ ſubjecto plano; ad lineam igitur eſt punctum I, [ſc. ad communem ſectionem
“ ſuperficiei a recta HL deſcriptæ, cum plano prædicto quod per BC tranſit]
“ ergo eſt punctum E ad lineam [quæ ſc. formatur ducendo perpendiculares
“ a linea in qua verſatur punctum I, ad ſubjectum planum.] Hoc quidem
“ univerſe reſolutum eſt, [ſc. quæcunque fuerit data ratio EF ad DC.] Si
“ autem hæc ratio æqualis fuerit rationi quam habet BA ad circumferentiam
“ quadrantis ADC, prædicta linea [in qua eſt punctum E] quadratrix efficitur.”

The 29th Prop. next following is, “ Poteſt etiam illud per lineam ſpiralem in plano deſcriptam reſolvi ſimili ratione.”—It is unneceſſary in this place to give the inveſtigation, but I ſhall remark ſome of the moſt material corrections of it by Dr. SIMSON.

In line 6. fol. 59. a. COMMAND. “ in ſuperficie cylindrica, in qua eſt linea “ ſpiralis.”—Dr. SIMSON ſuppoſes from Note F on this Propoſition, that in the MS. uſed by COMMANDINE the reading was ἐν κυλινδροέιδει ἄρα ἐπιφανείᾳ, which is more correct, and is alſo confirmed by the MS. BULL. It ſhould therefore have been tranſlated “ in ſuperficie cylindroeide.”—Alſo, inſtead of "in qua eſt linea ſpirali," it ſhould be "quæ formata eſt a linea ſpirali:" for in MS. Bull it is "τῇ ἀπὸ τῆς ἑλίκος," which is plainly the true reading.

Lin. 8. "ad lineam eſt ipſum K," add, "ſc. ad eam lineam quæ eſt "communis ſectio ſuperficiei cylindroeides et ſuperficiei conicæ." Lin. 11. for "ergo K et I ſunt in ſuperficie," read, "ergo eſt punctum I in ſuperficie, ſc. "in ea quam vocavit ϖληκτοείδος." Lin. 13. for, "ad rectam igitur lineam eſt "punctum I;" read, "ad lineam igitur eſt punctum I,* "ſc. eam quæ com- "munis eſt ſectio ſuperficiei πληκτοείδεος† cum plano quod eſt ſuper rectam BC, "et ad ſubjectum planum eſt inclinatum per dimidium recti."

Note H. p. 65.

For the ſatisfaction of ſome readers, I ſhall inſert Dr. Simson's Paper on Series, encloſed in his letter to Dr. Jurin, of Feb. 1, 1723, (for which ſee note p. 65.) The two Preliminary propoſitions are very eaſy, and are now to be found in all the compleat treatiſes of trigonometry; but as they are ſhort, they are inſerted as they ſtood in Dr. Simson's original paper.

Dr. Simson about a month after received Dr. Jurin's anſwer, (of March 5, 1723,) communicating Mr. Machin's ſeriefes; and to his paper he has added a note, ſtating the correſpondence of five of his own ſerieſes with thoſe of Mr. Machin's. Mr. Machin communicated ſeven; and two of them, which had not before occurred to Dr. Simson, were readily inveſtigated by him.

"Prop I. Fig. 11. Sint in circulo cujus centrum eſt C duo quilibet arcus AB, "AD, qui ſimul ſumpti ſunt quadrante BL minores; erit exceſſus quadrati quod "fit a ſemidiametro ſupra rectangulum ſub tangentibus arcuum AB, AD ad "quadratum a ſemidiametro, ut tangentes arcuum AB, AD ſimul ſumpti, ad "tangentem ſummæ arcuum.

"Sint enim AE, AF tangentes arcuum AB, AD, et ſit BG, tangens ſummæ "ipſarum BD, et juncta CL, occurrat ipſi AF in H, ducatur vero FK parallela "ipſi AC, et occurrens EC in K, et jungatur KH. Igitur quoniam anguli "HCK, HFK recti ſunt, erunt puncta H, F, C, K, in circulo; ergo angulus "FHK equalis eſt ipſi FCE, et proinde ſimilia ſunt triangula rectangula

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† For ſome obſervations on πληκτοέιδος, ſee Appen. II.

* This is confirmed by MS. Bull, which has πρὸς γραμμῆς ἄρα τὸ ι.

“ HFK, CBG. Eſt autem ratio EF ad BG compoſita ex rationibus EF ad FK “ et FK ad BG; *i. e.* ex rationibus EA ad AC, et FH ad CB ſeu AC; ergo EF “ eſt ad BG ut rectangulum EA in FH, ad quadratum ex AC; ſed eſt EA in FH “ exceſſus rectanguli EAH, *i. e.* quadrati ex AC, ſupra rectangulum contentum “ tangentibus EA, AF, et eſt hic exceſſus ad quadratum ex AC ut EF ad BG. “ *Q. E. D.*

“ Cor. 1. Hinc ſi arcus AB, AD, æquales fuerint, erit exceſſus quadrati a “ ſemidiametro ſupra quadratum tangentis arcus cujuſvis octante circumferentiæ “ minoris ad quadratum a ſemidiametro, ut tangens duplicatus ad tangentem “ arcus duplicati.

“ Cor. 2. Hinc etiam inveniri poteſt tangens arcus qui multiplex eſt “ arcus alterius, cujus tangens datur. Sit a tangens arcus A, et erit tangens “ arcus $n \times A$.

$$\frac{na - \frac{n.n-1.n-2}{1.\ 2.\ 3.} a^3 + \frac{n.n-1.n-2.n-3.n-4}{1.\ 2.\ 3.\ 4.\ 5.} a^5 \text{ \&c.}}{1 - \frac{n.n-1}{1.\ 2.} a^2 + \frac{n.n-1.n-2.n-3}{1.\ 2.\ 3.\ 4.} a^4 \text{ \&c.}}$$

“ Numeratoris hujus ſeriei ſi n ſit numerus par, ſumendi ſunt tot termini “ quot ſunt unitates in $\frac{1}{2} n$; denominatoris vero tot quot ſunt unitates in $\frac{1}{2} n + 1$. “ Si vero fuerit n numerus impar, tam numeratoris quam denominatoris “ ſumendi ſunt termini $\frac{n+1}{2}$. Hanc ſeriem video apud Christ. Wolfium, “ ex ſinubus derivatum.

“ Prop. 2. fig. 12. Sint duo quilibet arcus AB, AD, quorum major non “ excedit quadrantem; erit quadratum a ſemidiametro una cum rectangulo “ ſub tangentibus arcuum ad quadratum ex ſemidiametro, ut differentia tangen- “ tium ad tangentem differentiæ arcuum.

“ Simili prorſus modo quo precedens demonſtratur.

“ Cor. 1. Hinc ſi arcus minor AB, æqualis fuerit dimidio quadrantis, erit “ radius una cum tangente majoris arcus ad ipſorum differentiam, ut radius ad “ tangentem arcus quo AD ſuperat dimidium quadrantis. Si vero arcus major “ AD æqualis fuerit dimidio quadrantis; erit radius una cum tangente minoris “ arcus, ad ipſorum differentiam, ut radius ad tangentem arcus quo dimidium “ quadrantis ſuperat arcum AB, *i. e.* in utroque caſu erit ſumma tangentium ad “ ipſorum differentiam ut radius ad tangentem differentiæ arcuum. Corolla- “ rium vero hoc, ni fallor, jamdudum innotuit.

" Præcedentium ope innumeræ ſeries exhiberi poſſunt ad longitudinem circumferentiæ inveniendam. Plurimæ vero, ad uſum paratiores, in ſequente " ſerie continentur. Sc. ſint tangentes $\frac{1}{a}$, $\frac{1}{2^n a}$ et ſit A arcus cujus tangens eſt $\frac{1}{a}$ " et erit, exiſtente radio = 1,

$$\left.\begin{array}{l} \frac{1}{a} - 1 \times \overline{\beta + 2\gamma + 4\delta + 8\epsilon} \text{ \&c.} \\ -\frac{1}{3} \times \frac{2^n}{\overline{2^n a}^3} + \frac{1}{3} \times \overline{\beta^3 + 2\gamma^3 + 4\delta^3 + 8\epsilon^3} \text{ \&c.} \\ +\frac{1}{5} \times \frac{2^n}{\overline{2^n a}^5} - \frac{1}{5} \times \overline{\beta^5 \times 2\gamma^5 \times 4\delta^5 \times 8\epsilon^5} \text{ \&c.} \\ -\frac{1}{7} \text{ \&c.} \end{array}\right\} = \text{Arcui A.}$$

$$\text{Eſt vero } \left.\begin{array}{r} \beta = \frac{1}{\overline{4aa \times 3} \times a} \\ \gamma = \frac{1}{\overline{16aa + 3} \times 2a} \\ \delta = \frac{1}{\overline{64aa + 3} \times 4^a} \\ \epsilon = \frac{1}{\overline{256aa + 3} \times 8a} \\ \text{\&c.} \end{array}\right\} \begin{array}{l} \text{Terminorum autem } \beta, \\ \gamma, \delta, \text{ \&c. tot ſumendi} \\ \text{ſunt, quot ſunt uni-} \\ \text{tates in numero in-} \\ \text{tegro } n. \end{array}$$

" Facile deducitur hæc ſeries ex Cor. 1 et 2. Prop. 1. et ex Prop. 2. Sit jam " arcus D datum habens tangentem, datamque rationem ad circumferentiam; " Sc. ſit $2\,r\,$D=circumf.$=c$; ſitque mA is arcus qui vel proxime excedit arcum " D, vel ab ipſo proxime deficit exiſtente m numero integro; et inveniatur " tangens arcus mA ope Cor. 2. Prop. 1. deinde inveniatur tangens differentiæ " arcuum mA & D ope Cor. 1. Prop. 2. Sit tangens hic t, et erit arcus cujus " eſt tangens, $t - \frac{1}{3}t^3 + \frac{1}{5}t^5$ &c. ut notum eſt; vocetur hic arcus B, et erit " $m\text{A} \mp \text{B} = \text{D}$, unde $rm\text{A} \mp r\text{B} = r\text{D} =$ circumferentiæ circuli cujus diameter " =1.

Ex. 1. Sit $a=4$, $n=0$, $r=4$, $m=3$, eritque $t=\frac{5}{99}$.

$$\text{Et} \quad rmA + rB = \overline{\frac{12}{4} + \frac{20}{99}} - \frac{1}{3} \times \overline{\frac{12}{4^3} + \frac{20^3}{99^3}} + \frac{1}{5} \times \overline{\frac{12}{4^5} + \frac{20^5}{99^5}} \text{ \&c.*}$$

N. B. Quoniam $\frac{1}{4}$ eſt tangens arcus A erit tangens arcus m A. *i. e.* 3 A $= \frac{47}{52}$ unde tangens arcus $45^\circ - 3$ A erit $\frac{5}{99}$.

Ex. 2. Sit $a=5$, $n=1$, $r=4$, $m=4$; eritque $\beta=\frac{1}{515}$; $t=239$.

$$\text{Et} \quad rmA - rB = \frac{16}{5} - \frac{16}{515} - \frac{4}{239} - \frac{1}{3} \times \overline{\frac{32}{10^3} - \frac{16}{515^3} - \frac{4}{239^3}} + \frac{1}{5} \times \overline{\frac{32}{10^5} - \frac{16}{515^5} - \frac{4}{239^5}} \text{ \&c. \&c. \&c.}$$

Ex. 3. Sit $n=0$ manentibus cæteris.

$$\text{Tunc} \quad rmA - rB = \frac{16}{5} - \frac{4}{239} - \frac{1}{3} \times \overline{\frac{16}{5^3} - \frac{4}{239^3}} + \frac{1}{5} \times \overline{\frac{16}{5^5} - \frac{4}{239^5}} \dagger$$

“ Et manifeſtum eſt ſeries has celerius convergere quo major eſt n, terminos “ vero ipſarum, propter eandem cauſam, complexiores evadere, et proinde plus “ laboris ad ipſos computandos requiri, ut ex duabus ultimo præcedentibus “ ſeriebus liquet.

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* “ This is Mr. MACHIN’s 4th, in the paper he ſent me down in Dr. JURIN’s Letter of “ the 5th of March 1723.”

† “ Quæ eſt ſeries optima a doctiſſimo Dom. MACHIN inventa.” Et eſt ejus 6ta.

Ex. 4. Sit $a=2$, $n=o$, $r=4$, $m=2$. Eritque $t=\beta=\frac{1}{7}$

$$\text{Et } rmA - rB = \frac{8}{2} - \frac{4}{7}$$
$$-\frac{1}{3}\times\overline{\frac{8}{2^3}-\frac{4}{7^3}}$$
$$\times\frac{1}{5}\times \&c.$$

This is Mr. MACHIN's 2d.

Ex 5. Sit $a=3$. Cæteris manentibus.

$$\text{Eritque } rmA + rB = \frac{8}{3}+\frac{4}{7}$$
$$-\frac{1}{3}\times\overline{\frac{8}{2^3}+\frac{4}{7^3}}$$
$$+\frac{1}{5}\times \&c.$$

This is Mr. MACHIN's 3d.

Ex. 6. Denique, ſit $a=2$, $n=o$, $r=4$, $m=1$, eritque $t=\frac{1}{3}$.

$$\text{Et } rA+rB=\frac{4}{2}+\frac{4}{3}-\frac{1}{3}\times\overline{\frac{4}{2^3}+\frac{4}{3^3}}+\frac{1}{5}\times\overline{\frac{4}{2^5}+\frac{4}{3^5}}\ \&c.$$

Quæ omnium eſt ſimpliciſſima. Eſt 1^{ma} Dom. MACHIN.

Similiter poſito $r=6$, derivantur aliæ ſeries, ut ſi ſit

$a=\sqrt{3}$, $n=1$, $m=2$. erit $t=\frac{1}{15\sqrt{3}}$.

$$\text{Et } rmA-rB=2\sqrt{3}-\frac{2\sqrt{3}}{5\times3}$$
$$-\frac{1}{3}\times\overline{\frac{\sqrt{3}}{2\times3}-\frac{2\sqrt{3}}{5^3\times3^4}}$$
$$+\frac{1}{5}\times\overline{\frac{\sqrt{3}}{2^3\times3^2}-\frac{2\sqrt{3}}{5^5\times3^7}}$$
&c.

“ Convergit hoc duplo celerius ſerie ſimplici ex tangente $\frac{1}{\sqrt{3}}$ derivata, ſed plus “ quam duplo labore.”*

Theſe few notes were added by Dr. Simson to his original paper, after he received Mr. Machin’s ſerieſes.

Above thirty years after this correſpondence, Dr. Simson gives ſome account of it to the late Earl Stanhope, in a letter dated Jan. 9, 1758, of which the following extract will not be unacceptable.

After acknowledging ſome ingenious communications of ſeries from Lord Stanhope, he adds: “ In the beginning of the year 1723 I ſent up to Dr. “ Jurin, Secretary to the Royal Society, a method of deriving ſeries for finding “ the circumference of the circle from the ſeries $1 - \frac{1}{3} + \frac{1}{5}$ &c. which by itſelf “ is of no uſe. I had before this deſired him to inform me if he knew if any “ application had been of this ſeries for finding of others that converged quickly; “ he wrote to me that he knew of none, and deſired me to ſend up the paper, “ which I did, and gave ſeven ſeries as examples of the method. He wrote me “ in anſwer, that on communicating it, he found that Mr. Machin had in the “ year 1705 or 1706, given in a paper to the Royal Society upon this affair, “ which he afterwards took back again; but that he had prevailed on Mr.

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* “ In the paper wrote and ſigned by Mr. Machin, which Dr. Jurin ſent me incloſed “ in his letter of the 5th of March 1723, there are ſeven ſerieſes ſet down for finding the “ arch of 45°, the

“ 1ſt of which is the ſame with the 6th in this letter; and the
“ 2d is the ſame with the 4th of this; and the
“ 3d is the ſame with the 5th of this; and the
“ 4th is the ſame with the 1ſt of this; and the
“ 6th the ſame that is here the 3d.

“ Mr. Machin’s 5th is for the arch of 45°. as all his others are, when radius is 1.

$$\frac{3}{4} + \frac{1}{20} + \frac{1}{1985} - \frac{1}{3} \times \overline{\frac{3}{4^3} + \frac{1}{20^3} + \frac{1}{1985^5}} + \frac{1}{5} \times \text{\&c.}$$

“ His 7th is $\frac{8}{10} - \frac{1}{100} - \frac{11}{5637} + \frac{10893}{41480222056636}$

$$- \frac{1}{3} \times \overline{\frac{8}{10^3} - \frac{1}{100^3} - \frac{11^3}{5637^3} + \frac{\overline{10893}^3}{41480222056636^3}} + \text{\&c.}$$

“ Vide the inveſtigation of the 5th and 7th ſeries (of Mr. Machin) in a MS. book.”

"Machin to ſend me a ſhort account of it, which he did in a paper ſigned "by him, which Dr. Jurin incloſed in his letter to me. It contained ſeven "ſeries, two of which were different from thoſe I had ſent, the reſt were the "ſame. In caſe your Lordſhip has not ſeen theſe ſeries, I ſhall ſet down two "of them."

Dr. Simson then ſtates the ſeries of Mr. Machin, No. 5, in the preceding note; and alſo one which he had ſent to Dr. Jurin, and was not returned by Mr. Machin, viz. Ex. 2d. in the preceding paper. Of this laſt he obſerves, "This ſeries is one of mine, and converges very quickly; and becauſe one part "of it conſiſts of powers of $\frac{1}{10}$, is raiſed with little more trouble than Mr. "Machin's, by the powers of $\frac{1}{5}$ and $\frac{1}{239}$, which was publiſhed in Mr. "Jones's *Synopsis*. It gives the circumference to the diameter 1." He adds, "The ſeries which Mr. Euler gives in page 107, vol. i. of his *Introductio in "Analysin Infinitorum*, (Lauſan. 1748,) is one of thoſe which I ſent up, and "which Mr. Machin ſent down, [being Ex. 6. before mentioned] but it does "not converge ſo quickly as the foregoing two." &c.

In the remainder of this long letter are ſome other obſervations on this and ſome other matters in the work of Euler referred to. But the inſertion of them is unneceſſary, ſince of theſe methods of ſquaring the circle by Mr. Euler, there is a full and explanatory detail, with many judicious remarks and improvements by Mr. Baron Maseres, in his great repoſitory of curious mathematical tracts, the *Scriptores Logarithmici*, vol. iii. p. 169, &c.†

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† It may be proper to refer the reader to ſeveral notices of theſe ſerieſes of Mr. Machin's, in Baron Maseres's *Eſſay on the Negative Sign*; alſo in his *Scriptores Logarithmici*, vol. iii. p. 157; and in Dr. Hutton's *Menſuration*. Dr. Hutton's explanation of Mr. Machin's ſeries, publiſhed by Mr. Jones, is reprinted in the ſame volume of *Script. Log.* with another tract alſo of the Doctor's on the ſame ſubject, p. 207, which had been publiſhed in the *Philoſophical Transactions*, and contains ſome curious diſcuſſions reſpecting ſuch ſerieſes, and ſome important improvements.

NOTE I. p. 69.

In a letter to Earl **STANHOPE**, of 22d of March 1751, he mentions very particularly the decline of his memory, and his inability to attempt ſome things recommended to him by his Lordſhip. "Perſons of my age (now paſt ſixty-"three) generally loſe the ability they had when younger, of a quick and ready "imagination; and their memory, (which, in my opinion, is either the imagi-"nation of ſenſations paſt, or the recalling imaginations we had formerly) "manifeſtly decays; and ſo far with me, that I have oftentimes difficulty to "recall thoſe I had the laſt hour, or even a few minutes before. And in long "inveſtigations, where it is neceſſary to look back a good way, this inability is "moſt eaſily obſerved, eſpecially when moſt of the ſteps are not wrote down; for I "remember ſince I could go through a longer ſeries of ſteps without writing, "than I can now well do with the help of it.* This, my Lord, makes me "afraid that I ſhall not be able to engage in the undertaking you are pleaſed to "recommend to me, and which, indeed, would be very agreeable to me; the "applying the method of the ancients to the modern inventions, ſo as they "might be demonſtrated in ſuch a way as would (to uſe your Lordſhip's juſt "and elegant deſcription of accuracy and ſtrictneſs) convince an **EUCLID**, an "**ARCHIMEDES**, or an **APOLLONIUS**, riſen from the grave, to examine "them. My ſcholars, Mr. **MOOR**, Mr. **WILLIAMSON**, and particularly "Mr. **STEWART** of Edinburgh, I hope may be able to do ſomething this "way; and I ſhall not fail to recommend it to them, and direct them as far "as I can."———"I hope, if **GOD** grant me health, I may do ſomething "towards reſtoring ſome pieces of the ancients; the firſt ſix, and the eleventh "and twelfth books of **EUCLID**'s *Elements* are near ready, excepting the "figures. Next to this, I have in moſt forwardneſs **APOLLONIUS** *de Sectione* "*Determinata*; but **PAPPUS**, though I have done a good deal towards reſtoring "him, is ſo large, that it frightens me to meddle with him."———Towards the end of this letter he adds, "And I hope to do **EUCLID** juſtice, if it pleaſe "**GOD** to give me opportunity to publiſh the eight books of his *Elements*, "which want of ſecurity from being pirated does yet delay."

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* I ſhall mention in this place an obſervation of Dr. **SIMSON** on mathematical enquiry, which, though not immediately connected with the ſubject of this letter, may gratify the

NOTE K. p. 72.

Among the ſmall remains of Dr. SIMSON's mathematical correſpondence which have been preſerved, are two letters from the late GEORGE LEWIS SCOTT, eſq; Commiſſioner of Exciſe, with copies of two from Dr. SIMSON in reply. The general object of theſe letters was mentioned in the memoir, and for the ſatisfaction of mathematical readers, the portions of them reſpecting the comparative merits of the ancient and modern analyſis are ſubjoined. Mr. SCOTT's firſt letter is dated London, 12th April 1764, and after ſome obſervations on Dr. SIMSON's edition of the *Data*, which the Doctor had ſolicited from him, and in which he diſcovers ſome prejudice againſt that book, thinking its uſe to be rather logical than mathematical, he adds:

" To be more particular, I obſerved that the 27 firſt Propoſitions relating
" to magnitudes and their ratios merely, might be much more conciſely treated,
" and receive an additional evidence from the algebraic method. I ſhall give
" but one inſtance from Prop. xvii. which is the xith in BARROW, and is
" by him left in darkneſs; you have given it light; but would it not be ſhorter
" and clearer to ſtate it and its demonſtration in ſymbols?

" The firſt hypotheſis, that $x - a : y :: m : n$ where a, m, and n, are data.
" The theſis that $x -$ dat: $x + y$, in ratione data. The hypotheſis may be
" changed into

" $x - a : y :: a : b$ therefore $x - a + y : x - a :: a + b : a :: a : \frac{a^2}{a+b}$.

" Therefore by 12 . 5th Elem. $x - a + y : x - a :: x + y : x - a + \frac{aa}{a+b}$

" and $x - a + y : x - a :: a + b : a$

" Therefore $x - a + \frac{a^2}{a+b} : x + y :: a : a + b$. q. d e.

" The ſecond hypotheſis is that $x - a : x + y :: a : b$. which may be ſhewn in
" like manner.

reader. It is alſo in a letter to Earl STANHOPE, April 9, 1753. "What your Lordſhip
" ſays of the uſefulneſs, and often the neceſſity, of uſing the method of induction, I have
" frequently had the experience of; and though it be a proof of the weakneſs of the human
" mind, it is at the ſame time a good help to the finding out whether a Propoſition be true or
" not, and moſt powerfully excites us to ſearch after a ſtrict demonſtration."

" I have followed your reaſoning; mine is eſſentially the ſame; but is not " the diſtinction by ſymbols of the indeterminate from the determinate or " given quantities or magnitudes concerned in the queſtion of uſe? and is not " the conciſeneſs of the expreſſion of great uſe alſo? Many of the Propoſitions " of the data might be thereby reduced to intuitive evidence. In this particular " demonſtration, it is ſhewn at once, what magnitude is to be taken from x " (viz. $a-\frac{a^2}{a+b}$) that the remainder may be in a given ratio to $x+y$. I mention " theſe particulars only to introduce a more general obſervation, which is, " that wherever magnitudes and their ratios are conſidered, the method of the " ancients ſeems inferior to that of the moderns, with reſpect to facility, though " not at all ſo in point of accuracy. When I look into **Apollonius**, I " admire the great man; and I may ſay the ſame of **Gregory** of St. Vincent " and of **Huygens**; but who can help complaining of the tedioufneſs of their " demonſtrations? They ſeem to me in many caſes like a man who would " reject the Indian ſymbols of number, and perform all the arithmetical " operations in words at length. This might be done; but to what good " purpoſe? When I ſay to what purpoſe, I mean geometrical purpoſe. Otherwiſe " I much approve of the elements being demonſtrated in words at " length, becauſe of their logical uſe. For in the affairs of life we have no " ſymbols, but common words; and it is of ſervice to habituate the mind, to " accurate and diſtinct expreſſions, and even to formal ſyllogiſms."

In the ſame letter after, mentioning the Propoſitions of the *Data* from the 27th to the 62d, as very eaſy, he adds: " In theſe Propoſitions I ſee " little or no room for algebra; and indeed, in queſtions of poſition, it often " comes in aukwardly enough. Dr. **Wallis**, one of the great patrons of " this method, owns it. It is therefore a proper enquiry to determine, if " poſſible, when one method is to be uſed, and when the other. You ſeem to " promiſe ſomething of this kind in the preface to your *Conics*. I wiſh you " could find reſolution to execute what I am perſuaded is almoſt ready in your " manuſcripts. **Thomas Simpson**, in his *Exercises*, ſays, that ſometimes " the algebraical, ſometimes the geometric method is preferable. I believe it. " But it ſeems to me that notwithſtanding the praiſes beſtowed by Sir **Isaac** " **Newton** and his followers on the method of the ancients, his own *Principia* " are but algebra diſguiſed, as **Machin** uſed to ſay. Though at the ſame

“ time he owned that he thought Sir Isaac's taſte of demonſtration preferable “ to Huygens's. I own myſelf of his opinion; as the only difficulty attending “ Sir Isaac's arguments, are the ſteps ſuppreſſed for the ſake of brevity.”

I ſhall add an extract from another letter of Mr. Scott's, dated 1ſt May 1764, as it is connected with the ſubject of the preceding.

When mentioning Dr. Simson's edition of the *Data*, he adds, “ I think “ that ſuch as would form themſelves upon the model of the ancients, will “ do well to ſtudy the *Data*. I even wiſh that I had been initiated in that way. “ But having once begun in the modern analyſis, I have not had leiſure to “ ſtrike into another road, eſpecially, as, to confeſs the truth, it ſeems to me “ much more tedious and more doubtful with reſpect to ſucceſs. I mean “ not that the method of the ancients leaves any doubt in the mind; but “ that the ſucceſs of attempting the ſolution of a difficult problem is more “ doubtful in the way of the ancients, than in that of the moderns; although, “ when the ſolution is once found, it be often true that demonſtrations formed “ according to the ancients are more agreeable, and more ſatisfactory to “ the mind, than thoſe derived from algebra. I ſay often, for it is far from “ being always ſo. To me a demonſtration derived from the conſideration “ of a multitude of ratios, and their compoſitions, and expreſſed in the “ manner of the ancients, is ſo far from being clearer than the algebraical “ method, that it ſeems vaſtly more difficult and obſcure. And I would “ appeal to any beginner, and aſk him, whether the general algebraic ſolution “ of the problem for determining the *foci* of Lenſes, or Huygens's method, “ be moſt clear. I might bring many other inſtances, where Propoſitions “ can be demonſtrated in the ſame manner as Euclid has done in his “ firſt four books, which often reduce the truth of a Propoſition to almoſt “ intuitive evidence. I admit that it would be in vain to look out for any “ aſſiſtance from algebra; and yet I think with Dr. Barrow, that moſt “ of the Propoſitions of the ſecond book are rather clearer when treated “ algebraically, than in Euclid's method. The perſpicuity of the fourth “ of that book has often been inſiſted on. It is very clear, undoubtedly; “ but ſtill our friend Dr. Matthew Stewart thinks that the original “ demonſtration was from the preceding Propoſitions, without a figure, “ which is entirely analogous to the algebraic method. The fifth book is “ not peculiar to geometry, but is equally applicable to quantity as it is to

“ magnitude; nor does the doctrine of proportion receive any evidence from “ its being expounded by lines. The ſymbols of algebra would, I think, be “ of uſe here. In the ſixth book there are many Propoſitions which may “ be treated either geometrically or algebraically, without any preference of “ advantage on either ſide: for wherever an analogy is uſed, it may as juſtly “ be ſaid to be algebraical, as it can be called geometrical. I may demonſtrate “ geometrically the 47. El. 1. from the doctrine of ſimilar triangles, and I “ may do the ſame algebraically. The two demonſtrations differ only in ſtile; “ and GUISNEE is miſtaken, when he ſays this theorem cannot be demonſtrated “ algebraically. I readily admit that the theorems about the congruity and “ ſimilitude of triangles cannot be demonſtrated by algebra, and that theſe “ are the true geometrical elements; which being eſtabliſhed, the other Pro- “ poſitions in the *Elements* may be derived, either by the method of the “ ancients, or by that of the moderns, equally well; at leaſt, if the ancient “ method has the advantage in eaſy queſtions, that of the moderns ſeems “ of almoſt indiſpenſible neceſſity in more difficult enquiries. I admit, however, “ that many parts of your writings, and of Dr. MATTHEW STEWART's, might “ furniſh ſtrong objections to what I advance, nor do I pretend to anſwer “ ſuch objections: all I ſay at preſent is, that I know a perſon very well verſed “ in both ancient and modern geometry, who ſeems diffident of his own “ abilities to comprehend Dr. STEWART's determination of the Sun's diſtance. “ He and I would, I believe, find the ſame difficulties in Dr. STEWART, “ that BULLIALDUS did in ARCHIMEDES. And to conclude the ſubject, “ I muſt confeſs that I cannot find that ſuperiority of evidence in the method “ of the ancients, that you and many other able geometers, in this iſland, “ contend for. Their great genius and accuracy I do ſee, and I ſee ſometimes “ the precipitation and innacuracy of modern geometers. But theſe are the “ faults of individuals, and not of the art.”

In the ſame letter, after ſome remarks on the data not connected with the particular ſubject of the preceding extract, he adds, “ In your notes you “ bring two inſtances of the uſe of the data, and think that the conſtruction “ of the laſt problem could not be derived from an algebraical ſolution. “ I find, indeed, that the algebraic ſolution is more complicated than one “ would at firſt ſuſpect, on reading the problem. Even in very ſmall numbers, “ the ſolution brings you to large ones. Thus, if $x : y :: 6 : 7$, alſo $y-1 : z :: 3 : 4$

“ and $xx + zz = 100$. I find $x = \frac{168}{277} \pm \frac{\sqrt{28224+2203812}}{277}$ and

“ $\frac{\sqrt{2232036}}{277} = \frac{1494}{277}$, therefore $x = \frac{168}{277} \pm \frac{1494}{277} = \frac{1662}{277} = 6$.

“ and $x = -\frac{1326}{277}$, therefore $y = 7$, and $-\frac{1547}{277}$

Alſo $z = 8$, and $-\frac{2432}{277}$, and both roots anſwer the

“ queſtion.

“ In the general ſolution, ſuppoſing

$$x : y :: a : b$$
$$y - a : z :: a : c$$
$$xx + z^2 = d^2$$

“ I find $\frac{bc}{a^2} x - c = z$, then taking $a : b :: c : f = \frac{bc}{a}$ and

“ $a : \sqrt{aa+ff} :: \sqrt{aa+ff} : h = \frac{aa+ff}{a}$. Alſo $h : c :: f : \frac{cf}{h} = p$. I have

“ $xx - 2px = \frac{a}{h}(dd - cc)$ and the conſtruction derived by this method is not “ ſo ſimple as yours. But the equation $xx + zz = dd$ being obviouſly a locus “ ad circulum, and the equation $\frac{bc}{aa} x - c = z$, a locus ad rectam, I find an “ eaſy conſtruction by combining them. For taking $a : b :: c : f = \frac{bc}{a}$ I have “ $\frac{f}{a} x - c = z$, which is very eaſily conſtructed; and then deſcribing a circle “ from the origin of the abſciſſa x with the Radius d, I have the other Locus, “ the interſection of which with the former determines the value of x. This “ is not leſs ſimple than yours; and you have alſo taken the advantage of the “ Locus ad circulum.——Hitherto there appears not to me any advantage in “ the ancient method. Nor do I ſee any from the firſt of the inſtances given “ at the end of your *Conics*. It is true your ſolution is more ſimple than “ NEWTON's, but your inveſtigation is not peculiar to ancient geometry. For “ what ſhould hinder an algebraiſt from ſuppoſing the angle BCD (in your figure) “ equal to BAC, and then calling BD $= x$, DC $= y$, and AB $= a$, we have by “ the ſimilar triangles, and by multiplying the extremes and means of the “ analogy, $ax + x^2 = y^2$, which is a Locus ad hyperbolam æquilateram, very "eaſily conſtructed. If Sir ISAAC's inveſtigation be more complex, it is not "becauſe he made uſe of algebra, but becauſe he did not make a convenient "election of terms. Now this election cannot be taught preciſely, though "Sir ISAAC himſelf has given many excellent obſervations relating thereto. "It is a ſagacity and habit to be acquired by the ſtudy of good models; and in "this reſpect there can be no doubt but the ſtudy of the ancients may be of "great uſe, even to an algebraiſt. I believe Mr. D'ALAMBERT ſpeaks truly, "when he ſays, that the ſtudy of geometry is perhaps too much neglected."

"After ſome other obſervations, he adds. "To conclude, I long much to "ſee a treatiſe of which you gave us hopes in the preface to your *Conics*, when "you ſay *in quibus autem differat analysis geometrica ab ea quæ calculo* "*instituitur algebraico, atque ubi hæc aut illa sit usurpanda alias disserendum*. "Such a treatiſe is much wanted; and I am perſuaded that you muſt have it "almoſt ready for the preſs among your papers, ſo that the labour of publiſhing "would not be great."

Theſe letters, it is obvious, were written by Mr. SCOTT without any intermediate letter from Dr. SIMSON. I ſhall ſubjoin the material parts of the Doctor's two anſwers, which he diſpatched ſeparately, though he mentions that he had received the ſecond before he had finiſhed his anſwer to the firſt.

Dr. SIMSON's firſt letter, diſpatched July 27, 1764.

"I am now, by the weakneſs of my memory and of my eyes, in no good "condition to write letters of any length; but yet I ſhall try to anſwer the "ſeveral particulars you mention, to the beſt of my ability. I am glad the "Problems have come to your hands. You perhaps have not yet leiſure to "read them, elſe I would have expected your opinion about them. Your "cenſures and corrections I value more than commendations. In the preface "to the *Data* it is ſaid, that they are of the moſt general and neceſſary uſe "in the ſolution of problems of every kind; and whoever tries to inveſtigate "the ſolutions of problems geometrically, will ſoon find this to be true; for "the analyſis of a problem requires that conſequences be drawn from the "things that are given, until the thing that is ſought be ſhewn to be given. "Now ſuppoſing that the book of the *Data* were not extant, theſe conſequences "muſt be found out and demonſtrated from the things given; which, in moſt

“ caſes, would take no little time and pains. But now having that book, the
“ Propoſitions of it need only be cited, or ſuppoſed to be known; for though
“ the ancients did not cite the particular Propoſitions either of Euclid's
“ *Elements*, or of his *Data*, yet they ſuppoſed them to be known, as is evident
“ from the ſolutions of problems given by Pappus, in which he always makes
“ uſe of the Data as being known to every geometer. The Elements are
“ neceſſary in every geometric reaſoning, whether in demonſtrating theorems
“ or in reſolving problems: and in the analyſis of problems the Data are of
“ as general and neceſſary uſe.”

“ The demonſtration of the 17th Dat. which you gave in ſymbols, is very
“ right; the ſhortneſs of the expreſſion by ſymbols is no doubt of uſe, but
“ then there is greater hazard of committing miſtakes by uſing them, than
“ when the words at length are made uſe of; and I believe that Dr. Barrow's
“ miſtake in this Propoſition has been owing to this ſhort-hand writing, which
“ coſt more time to underſtand, than if he had uſed the ordinary way. As A
“ is with him the given magnitude, and $\frac{B}{C}$ the given ratio, it is plain that
“ $A+B$ is the firſt of the magnitudes mentioned in the enunciation; from
“ which taking A, the remainder B has to C, the ſecond magnitude, a given
“ ratio; and the thing he ſhould have demonſtrated is, that $A+B$ is major
“ $A+B+C$ data quam in ratione, which he has not done, but only ſhewed
“ that $A+B$ is major $B+C$ *d. q.* in *r.* Many ſuch miſtakes are obſervable
“ in problems done algebraically; and in the 31 Prop. lib. i. of the *Loci plani*
“ p. 96, I have taken notice of a remarkable one in Schooten's *Exercitationes*,
“ p. 249. The given magnitude AE (in your notation $a-\frac{a^2}{a+b}$,) which
“ is to be taken from AB (or x) is eaſily ſeen to be the exceſs of AD (or a)
“ above ED, which is the 4th proportional to DC, DB and AD the given
“ magnitude; and if the given ratio of DB to BC be a to b, as you make it,
“ then the ratio of DC to DB will be that of $a+b$ to a.

“ You ſeem to think, that wherever magnitudes and their ratios are con-
“ ſidered, that the algebraical method is better than that of the ancients in
“ reſpect of facility, though not of accuracy. But what if the conſtruction
“ deduced from the firſt method, be nothing ſo good, either in reſpect of
“ ſhortneſs or elegance, as that from the other? which is often the caſe.

" You may at leiſure try the algebraic ſolution of Prob. 2. in pages 465 and " 466 of the Engliſh Euclid, which will furniſh you with an inſtance of this.

" The length of the demonſtrations in the method of the ancients ariſes, " for moſt part, from the ſtrict regard they had to accuracy; yet I do not " ſay that none of them can be made ſhorter; for ſometimes, by making " uſe of a more proper medium of the demonſtration, they can be ſhortened: " an inſtance of which you have in the 20th Prop. b. ii. of Apollonius's " *Conics*; which, by making uſe of the properties of the Aſſymptotes inſtead of " thoſe of the Diameters, is made ſhorter in the 40th Prop. of b. iii. of the " book which I publiſhed ſometime ago. I expreſſed myſelf indiſtinctly " in the preface to that book, which has made ſome think I deſigned to " publiſh ſomething, to ſhew when the ancient and when the modern method " ought to be uſed; but I meant to ſay nothing more than that it was not " proper to be done in that preface."

" I cannot gueſs on what account Mr. Machin, to whom you aſſent, thought " Sir Isaac's taſte of demonſtration preferable to that of Mr. Huygens. " Though I have the greateſt veneration for Sir Isaac's profound and penetrating " genius, and his moſt valuable and uſeful diſcoveries, yet I cannot but prefer " Huygens's elegance and perſpicuity, eſpecially in his *Horologium oscillatorium*, to Sir Isaac's; nor do I think the firſt has fallen into ſo many " miſtakes as the other, in things purely mathematical."*

I ſhall ſubjoin ſome extracts from Dr. Simson's letter in anſwer to Mr. Scott's 2d letter, of May 1, 1764.

" Dear Sir,

" Your approbation of the edition of the *Data*, with which you begin your " ſecond letter of May 1ſt, gives me great ſatisfaction. You obſerve that the " demonſtrations made by a multitude of ratios and their compoſitions, expreſſed in the manner of the ancients, are more difficult and obſcure than " the algebraic method. I confeſs a multitude of them makes a demonſtration difficult in any of the methods, and I found it was not only difficult to " my ſcholars to underſtand the 20th Prop. of Huygens's *Dioptrics*, but " uneaſy to me to prelect it to them, by reaſon of the great number of ratios " that are conſidered in the demonſtration, where there is only a ſtraight

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* Sir Isaac Newton himſelf entertained a high opinion of the elegance of Huygens's taſte in geometry. See Pemberton's View, Preface.

" line and its parts to aid the memory: and on this account I contrived a " demonſtration by help of a figure, by which the compoſition of ratios is " avoided, and nothing but the properties of parallel lines, and ſimilar " triangles, enter into; it which made it much eaſier both to my ſcholars and me."

" No doubt the fourth of the ſecond book of the *Elements* might have been " ſhewn from the third and the ſecond of it, and EUCLID, or even a far leſs able " geometer, could not but ſee it; but he preferred the way by figures in all the " demonſtrations of the ſecond book, as ſhewing the equality of the ſpaces " compared to the eye, which is more clear than when it is perceived by " the imagination. As to the fifth book, I do not ſee that the demonſtrations " in it can receive any help from algebra; and the ſtraight lines made uſe of " in it make the demonſtrations clearer and eaſier, than they would be without " them. I do not underſtand your meaning, when you ſay any analogy " may be called algebraical as well as geometrical. The expreſſing lines by " a ſingle letter does not make analogy or any thing elſe algebraical, any " more than when they are expreſſed by the two letters at their ends, nor do I " think any thing can be called algebraical, where no operation peculiar to " algebra is made uſe of. I ſhould be glad to ſee an algebraical demonſtration " of ſome propoſition in the 5th book, *ex. gr.* of the 17th, by which perhaps " I might underſtand your meaning. The 47th Prop. of 1ſt B. cannot be done " by ſimilar triangles in the place where it now ſtands; and when it is done " by them, I do not ſee that the demonſtration has the leaſt dependence " on any thing in algebra. You think algebra almoſt indiſpenſibly neceſſary " in difficult enquiries: I have, on the contrary, been obliged to make uſe of the " ancient method in many problems, in which I could not find that algebra was " of any uſe to me; ſuch were the two problems, the ſolutions of which I ſent " ſome time ago; and ſo are many others, of which the following is one. *Three " points and three straight lines being given in position, it is required to place " a triangle so that the three angular points may respectively be on the three " given straight lines, and the three sides pass through the three given points.*"*

" The ſolution which you give of the laſt problem at the end of the *Data*, " by help of two *Loci*, is very good; but I am not yet perſuaded, that the con-

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* The enunciation in this letter has reference to a figure which is here omitted. It is alſo to be remarked that in the *Opera Reliqua*, p. 388, is a ſolution of a caſe of this problem, viz. when the three given points are in one ſtraight line.

“ ſtruction ariſing from it, or the demonſtration of it, will be quite ſo ſimple “ as thoſe given in that place: if you had leiſure to write them out, without “ making uſe of any algebraical notation or operation, they might then be “ better compared. You think I have taken advantage of the *Locus ad Cir*“ *culum*, but I aſſure you it never came into my mind: the 34th Prop. in “ the *Data*, as you will ſee by looking to it, does indeed require the deſcription “ of a circle, by which the poſition of the ſtraight line AD (in Prob. p. 465,) “ is found.

“ It would take up too much time, and writing too much for my eyes, to “ ſhew the advantage of the ancient method above the algebraic, and how the “ precepts given in this laſt method, particularly in the *Arithmetica Universalis*, “ lead thoſe who obſerve them, from the right ſolution of geometrical “ problems, into ſuch as are quite out of the natural method; many inſtances “ of this occur in that book, among which is that problem which is the “ firſt at the end of the ſecond edition of the *Conics*. You ſay the ſolution “ there given is not peculiar to the ancient geometry, and you add, ‘what ſhould “ hinder an algebraiſt to ſuppoſe the angle BCD in my figure equal to BAC “ &c.?’ True, an algebraiſt may give a geometrical ſolution of a problem, but it is “ not therefore an algebraic ſolution. The ſolution you give in the algebraic “ notation depends nothing on algebra, for the *Locus* $ax+x^2=y^2$ flows from “ one of the firſt properties of the hyperbola demonſtrated in the *Conics*; “ and is the very ſame which is made uſe of in the 1ſt Prob. of the appendix. “ It may be ſaid that algebraiſts are blameable, not ſo much in making a wrong “ choice of a medium, as for hindering themſelves from making any choice “ not pointed out by their rules; one of which almoſt conſtantly made uſe “ of by them is, to let fall a perpendicular, that by help of the 47. 1ſt Elem. “ they may get an equation, which otherwiſe they could not ſo eaſily obtain.*

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* An obſervation of Des Cartes tends to confirm ſome of Dr. Simson’s remarks in this letter; for which ſee Branker’s Algebra, p. 65, (1668.) Des Cartes, in a letter “ not yet printed, writes thus: *In searching the solution of geometrical questions, I always* “ *make uſe of lines parallel and perpendicular as much as is possible; and I consider no other* “ *theorems but these two.* (The ſides of like triangles have like proportions,) *and* (in “ rectangle triangles the ſquare of the greateſt ſide is equal to the ſquare of the other two “ ſides,) *and I am not afraid to suppose many unknown quantities, that I may reduce the* “ *proposed question to such terms, as that it depends on no other theorems but these two.*” This letter of Des Cartes, however, was afterwards printed, Amſt. 1683, Epiſt. 72, part iii.

“ I forgot, when mentioning the laſt problem, at the end of the *Data*, to “ obſerve that in the equation you firſt bring it to, you give the value of “ what is called the negative root, viz. $x = -\frac{1326}{277}$.† It is one of the abſurdities introduced into algebra in the laſt age, to ſuppoſe every equation has “ as many roots as there are units in the index of its higheſt power, and “ conſequently that every quadratic equation has two: but the contrary, in “ the preſent caſe, can eaſily be ſhewn. Your equation is of the ſame form “ with this $x^2-4x=12$, the root of which you know is $x=\sqrt{12+4}+2$ or 6. “ And that this equation can have no other root, *i. e.* that there can be “ no number but 6, from the ſquare of which taking the quadruple of the “ number, the remainder will be 12, appears thus. Suppoſe it poſſible, that “ y is alſo the root of the equation, or that $yy-4y=12$. Then y muſt be “ either greater or leſs than 6 or x; (if any deny this axiom, I will reaſon “ no further with him on this matter;) ſuppoſe that it is greater, $y-4$ is greater “ than $x-4$, and therefore the product of y by $y-4$ is greater than the product “ of x by $x-4$; that is, $yy-4y$ is greater than $xx-4x$, and conſequently than “ 12. Therefore $yy-4y$ is not equal to 12; that is, y is not the root of the “ equation. The ſame thing follows, if y be ſaid to be leſs than 6.

“ *The paſſage you have ſet down from the preface to the Conics, might,* “ *I confeſs, make any one think that I deſigned to have publiſhed ſomething* “ *on the ſubject there mentioned; but indeed, as I wrote in my laſt, I blundered in the expreſſion, and have no papers, and never wrote any about* “ *that matter. R. S.*”*

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page 296. The ſentences ſucceeding the foregoing extract give additional explanations of his opinion, to which the reader is referred. I may alſo remark, that in this letter, (Ep. 72.) and in the following, (Epiſt. 73.) ſome difficulties are mentioned in the algebraical ſolution of the caſe of the *tactions*, in which three circles are given.

† Dr. SIMSON's notions about negative quantities in algebra have been frequently mentioned. His comment on the negative root of a quadratic equation, mentioned in Mr. SCOT's letter, though not connected with the other matters in this correſpondence, is retained as a characteriſtic ſpecimen, in his own words, of the Doctor's manner of treating that ſubject. He made no ſecret of his opinions reſpecting it: they were well known to all his contemporaries; and whatever objections may be made againſt them, he remained firm in maintaining them.

* This letter was diſpatched to Mr. SCOT, 13th Auguſt 1764; and no trace of correſpondence between theſe gentlemen, of a ſubſequent date, remains.

With reſpect to this correſpondence mathematical readers will form their own opinion; but it is proper to remark, that Dr. SIMSON, at the time of writing his letters, was in his 77th year. I venture to add a few remarks, which, to thoſe who have not conſidered the ſubject particularly, may be acceptable.

In all thoſe geometrical inveſtigations, in which the chain of argument can be rendered by the ancient analyſis both intelligible to the underſtanding, and pleaſing to the imagination, Dr. SIMSON's reaſoning in its favour will not be objected to. It muſt be admitted alſo, as he ſtates, that unleſs operations peculiar to algebra be introduced, an inveſtigation, though clothed in the characters of algebra, may yet be properly geometrical. He gives an example of the ſolution of a problem by the combination of two local equations; and ſuch may always be regarded as purely geometrical, when the *Loci* employed have been demonſtrated in known geometrical works, and are eaſily deducible from the *Data* in the problem without any algebraical operation. But even when an algebraical proceſs is requiſite for inveſtigating the *Local Equations*; or when, in the inveſtigation of a geometrical problem by algebra, ſome inferences may be more immediately drawn by means of known geometrical propoſitions than by the operations of that art; the algebraical ſolutions may by ſuch aids be much abridged, and often rendered much more elegant.

It is further to be remarked, that when geometry comes to be employed in practical menſuration, or in phyſical enquiries, the moſt obvious and moſt expeditious methods of inveſtigation, whether geometrical or algebraical, will uſually be preferred. It is well known that in ſuch applications, in general it is uſeful, and often abſolutely neceſſary, to expreſs concluſions (in whatever way they may be obtained) arithmetically; and hence it is frequently expedient to inveſtigate by algebra ſuch concluſions, though they might without difficulty be obtained by geometrical reaſoning, as they muſt ultimately aſſume the arithmetical form in the practical uſes of them. I muſt alſo obſerve, that Mr. SCOTT's remark reſpecting the combination of ratios is perhaps not fully anſwered by Dr. SIMSON; and that the remark may alſo in ſome caſes be extended to any long ſeries of ſteps, by the combination and ſeparation of ſquares and rectangles ariſing from the ſegments of a ſtraight line, by the uſe of the Propoſitions of the iid b. of EUCLID, or of others founded on that book. Though a demonſtration may be regarded as ſtrictly geometrical, when ſuch ratios, and the combinations of them, are ſet down either in words at length, or in a kind of ſhort-hand, by uſing ſome algebraical characters; yet when the mind loſes the diſtinct perception of the particular geometrical magnitudes compared, the evidence is ſimilar in its impreſſion to that of algebraical reaſoning, in which the previous demonſtration of the rules employed is the ground of our aſſent to the truth of the concluſions, and not the immediate perception of the geometrical magnitudes and their relations.—An analyſis or demonſtration in which many combinations of ratios are employed, may generally indeed be much ſhortened by admitting multiplication, diviſion, or other operations peculiar to arithmetic or algebra; but the inveſtigation or demonſtration then becomes truly algebraical. It muſt however be admitted, in the particular caſe here ſuppoſed, that the evidence of the algebraical proceſs is not materially different, or rather that the impreſſion of that evidence on the mind, is not very different, from that of the geometrical method. The ſame obſervation may be extended alſo to the caſe, when many combinations of ſquares and rectangles from the ſegments of a ſtraight line neceſſarily make a principal part of a geometrical analyſis or compoſition.

Dr. SIMSON acknowledges the difficulty of purſuing ſuch long trains of reaſoning, by combining the ratios in the Propoſition of HUYGHENS particularly alluded to. He found it even expedient to contrive a new demonſtration by figures, which might facilitate his own labour in lecturing, and more effectually keep alive the attention of his ſcholars, by preſenting objects better calculated for that purpoſe, and alſo for aiding the memory in retaining the long ſeries of ſteps in that Propoſition. But in ſuch Propoſitions an algebraical deduction would in many caſes be not leſs ſatisfactory, and far more expeditious, than any other method.

It muſt be conceded, however, by the patrons of the algebraical analyſis, that the ancient method, where it can be applied, with very few exceptions, is more elegant, and gives a more immediate and more pleaſing conviction of the truth of a geometrical propoſition, than the other. It muſt be admitted, alſo, that there are many geometrical problems, even in the claſs denominated *plane*, of which the algebraical ſolution is far from being obvious, and is often more difficult than the geometrical; even from the complexity of the calculation, without regarding the inferiority of the conſtructions which generally reſult from the ſolution of equations. Dr. SIMSON, in one of his letters to Mr. SCOTT, mentions an example; and others equally illuſtrative of the point might eaſily be pointed out.*

That many geometrical problems are more eaſily reſolved by algebra than by the ancient analyſis, there is no doubt; but when the object is to obtain a ſolution, in which rigid accuracy is combined with perſpicuity in every ſtep of both the analyſis and demonſtration, the ancient method is indiſputably to be preferred. But the diſtinguiſhing value of the modern analyſis, though not particularly conſidered in this correſpondence, is the means which it affords of reſolving geometrical problems, inacceſſible to the ancient analyſis in its preſent ſtate; and of applying geometry to phyſical inveſtigations, equally beyond the reach of that more elegant, though leſs powerful inſtrument. Such, indeed, has been the ſucceſs of the modern analyſis, in the vaſt enlargement of ſome important branches of ſcience, and of the arts depending on them, that all reaſonings on any theoretical imperfection in its principles, which does not affect the truth of the concluſions, is ſuperſeded by the wonderful extent and utility of its power and application.†

In a miſcellaneous MS. volume of the year 1722 were found, annexed to the ſolution of a geometrical problem, ſome ſhort obſervations by Dr. SIMSON on the ancient analyſis; which, though apparently ſuggeſted merely by that ſolution, and written without any particular view, yet mark the accuracy of his notions reſpecting that analyſis at this early period, before he had made much progreſs in the reſtoration of the loſt analytical works of the ancient geometers. They may however, notwithſtanding theſe circumſtances, be conſidered as a ſuitable addition to the Doctor's letters on the ſubject in his old age, which have been

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* The problem, (117 lib. vii. PAPPI) and the extenſions of it, of which a ſhort hiſtory is given in note E, may alſo be mentioned as examples.

† Such is the power of the modern analyſis in many complex inveſtigations, that an obſervation of WOLFIUS (tom. v. Math. Univ. p. 210) will not be conſidered as altogether extravagant, even by the rational admirers of ancient geometry. "Sane ſi "ARCHIMEDES et APOLLONIUS noſtro ævo reviviſcerent, in ſtuporem raperentur viſis "inventis recentiorum, quæ per algebram fuerunt in apricum producta; neque unquam "ſibi perſuaſiſſent patere ad talia mortalibus aditum." The original and powerful genius of ARCHIMEDES however, had his life been prolonged, might have anticipated many of the great diſcoveries of modern times; and have adorned them alſo with his elegant and accurate manner of demonſtration.

given in this note. To prevent unneceſſary trouble, the particular references to the problem are omitted.

“ Elegans eſt hoc exemplum veræ et genuinæ tam analyſios quam conſtructionis et demonſtrationis, methodo veterum inſtitutarum, quaſque algebra recentiorum nunquam præbuiſſet.”

“ In genere notandum eſt, analyſim eo puriorem et elegantiorem habendum eſſe, quo minus artificii in conſtructione analyſi præmiſſa invenitur; *i. e.* quo pauciores, eæque non niſi ob rationes obvias ducendæ lineæ, conſtructionem iſtam ingrediuntur.

“ Analyſes enim hac methodo inſtitutæ præferendæ ſunt iis quæ, quamvis breviores eſſe poterint, tamen ope conſtructionis minus obviæ perficiuntur, *i. e.* hæ ipſæ analyſes alterâ analyſi indigent, ut ad iſtas conſtructiones perveniatur, quæ propterea minus perfectæ ſunt cenſendæ. Perfectiſſimæ enim ſunt analyſes quæ a datis procedunt ad iſta quæ immediate cum iis connectuntur, atque ita donec ad quæſitum perveniatur. Verum quidem eſt, eos qui ſagaci et acri pollent ingenio, poſſe per pauciores gradus ad quæſitum, quaſi ſaltando, pervenire, ſed hoc nullo modo poteſt fieri, niſi vel animo perceperint grados omiſſos, quod redit ad methodum deſcriptam, vel tentando inciderint in conſtructionem quandam, a datis ſatis remotam, id quod omni cura ab analyſta eſt vitandum. Cum nihil verâ et genuinâ analyſi magis diſtat, nihil magis abhorreat, quam tentandi methodus; hanc enim amovere et certiſſima via ad quæſitum perducere, præcipuus eſt analyſeos finis; quamvis fatendum, tentando et in tenebris quaſi palpitando, quæſitum poſſe inveniri, ſed hac methodo nunquam, niſi cogente demum, ob deficientem analyſim, neceſſitate procedendum. De compoſitionis et demonſtrationis natura, cum eæ ex analyſi ſemper efficiuntur, eodem modo judicandum.

“ Porro animadvertendum eſt nihil in analyſi de quæſito vel ejus quavis affectione ſupponendum quod ex datis non clare fluat. *Ex gr.* In inveſtigatione loci alicujus non ſupponendum eſt locum eſſe *rectam* vel *circulum*, quoniam hoc eſt quod quæritur. Et hoc vitio laborat analyſis ex qua conſtructionem ſuam Prob. in Prop. 7, p. 25, lib. i. Apollonii *de Locis Planis*, hauſiſſe videtur Fermatius, nam inveniendo duo puncta in quibus locus quæſitus ſecat datas poſitione rectas, et affirmando rectam, hæc duo jungentem, eſſe locum quæſitum, videtur eum etiam in analyſi ſuppoſuiſſe, locum quæſitum eſſe rectam.

"Notandum denique in difficilioribus problematis tractandis multum ad "ſolutionem conducere, ſi a caſu aliquo ſimpliciore inciperimus."

He proceeds to illuſtrate this laſt obſervation by a particular caſe, which is too long for inſertion.

APPENDIX.

APPENDIX I.

Account of the Mathematical Collections of PAPPUS.†

§. I. *Two first Books of* PAPPUS.

PAPPUS, an eminent Mathematician of Alexandria, flouriſhed about the end of the fourth century.‡ In the very brief accounts which remain reſpecting him, he is mentioned as the author of ſeveral treatiſes all of which; except his *Mathematical Collections*, probably the moſt valuable of his writings, appear to have periſhed. This work, as the name imports is miſcellaneous, and beſides a variety of Propoſitions, both problems and theorems, contains ſome curious notices, not to be found elſewhere, of the hiſtory of mathematics and of mathematicians in his own and in preceding times. Its chief importance however is, as has already been mentioned, that from it alone we derive ſatisfactory information reſpecting the ancient geometrical analyſis. In the preface to the ſeventh book, particularly, is given a deſcription of the moſt valuable treatiſes in the collection, which in the celebrated ſchool of Alexandria* got the title of τόϖος ἀναλυομένος, and which was compoſed by ſome of the moſt eminent geometers of antiquity, for promoting the uſe of that analyſis.

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† In Dr. HUTTON's learned work, the *Mathematical Dictionary*, there is an excellent abſtract of the contents of the *Mathematical Collections*, to which I at firſt intended to have referred. But on further conſideration, I found that, for the purpoſe of giving a ſatisfactory account of Dr. SIMSON's commentaries and corrections of that work, a more detailed ſtatement was neceſſary, as will readily appear to the intelligent reader.

‡ See SUIDAS; (Πάππος,) alſo GER. VOSSIUS *de Chronologia Mathematicorum*; and MONTUCLA tom. i. In SUIDAS ſeveral works of PAPPUS are mentioned; but neither his Collections, nor a treatiſe alluded to by himſelf in b. iv. of the Collections, (COMMAND. fol. 56, 1588.)

Of the eight books of the *Mathematical Collections*, the firſt and about one half of the ſecond are preſumed to be loſt; the reſt have reached the preſent time, though with many imperfections, and in ſome paſſages ſo mutilated, that the meaning cannot be certainly aſcertained. The original Greek (except ſome ſhort extracts) has never been printed; and the only tranſlation of it, by Commandine, was firſt publiſhed at Piſaurum in 1588; and another edition, with little variation or improvement, was printed in 1660 at Bologna.† This tranſlation is accompanied with a commentary, often tedious, and in ſome places defective; but at the ſame time extremely valuable, from the explanation which it contains of ſome difficulties, and the correction of many errors in the MS. uſed by COMMANDINE, and which pervade all the MSS. of PAPPUS that have hitherto been examined. From COMMANDINE's manner of referring to the Greek, it appears that he had only one MS. for his guide. He died before the work had received his laſt corrections; and no account is given of the hiſtory or character of the MS. which he followed.[†] From a family diſpute between two ſons-in-law, the publication was ſuſpended for ſome time after his death; and at length, by the munificence of his patron the DUKE of URBINO, the tranſlation was printed, but confeſſedly without any correction whatever of the errors or omiſſions in the unfiniſhed work of COMMANDINE.[‡]

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* It is remarked by Vossius, in the before-mentioned work, "ab Euclidis tempore uſque ad tempora Saracenica, vix ullum invenire ſit nobilem mathematicum, quin vel patria fuit Alexandrinus, vel ſaltem Alexandriæ dederit operam Mathesi."

† Though the ſecond edition is ſaid to be corrected, yet this appears to be only in ſome preſs errors of the former edition, while new errors of the ſame kind are introduced: but there is no attempt to amend the obſcure and deficient paſſages in that tranſlation. From ſome convenience of the bookſeller, the original edition ſucceſ-

In this ſtate, however, it was a very intereſting communication to the mathematicians of that age, and excited much curioſity reſpecting ſome branches of the ancient geometry, which, before this publication, were little conſidered or known in modern Europe. Accordingly, not long after its appearance,

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ſſively got three title-pages and dates; the firſt, Piſauri 1588; the ſecond, Venetiis 1589; and the third, Piſauri 1602.

[†] COMMANDINE in many hundred notes quotes the CODEX GRÆCUS. In a very few places indeed he does quote the GRÆCI CODICES; but this laſt expreſſion ſeems to have ariſen merely from inadvertence, as in no inſtance is there any compariſon of readings of different MSS. though, in a great many caſes, from the ſenſe, he corrects the blunders of the one manuſcript to which he uſually refers.

[‡] See the Dedication, and alſo the addreſs to the reader, in which theſe circumſtances are mentioned; and it is alſo obſerved that COMMANDINE had made diligent ſearch for the two firſt books of PAPPUS, but without ſucceſs.

SNELLIUS, VIETA, MARINUS GHETALDUS, and FERMAT, attempted, from the deſcription of the τίϖος ἀναλυομένος in the ſeventh book, to reſtore ſeveral of the loſt books of APOLLONIUS, which made a principal part of that collection. Afterwards SCHOOTEN, Dr. WALLIS, and ſubſequently Dr. HALLEY, purſued the ſame track; and thus a number of theſe ancient treatiſes were reſtored, with various ſucceſs indeed, and with different characters of elegance and accuracy, as has already been obſerved in the foregoing Memoir.

The two firſt books of PAPPUS are not in COMMANDINE's tranſlation, from their not being found in any of the MSS. to which he had acceſs; and they are wanting alſo in ſeveral other MSS. remaining in various libraries of Europe.* In a MS. of PAPPUS, in the Savilian Library at Oxford, (No. 9,) there is preſerved about one half of the ſecond book, which was publiſhed by Dr. WALLIS in 1688, with a Latin tranſlation, and valuable notes, explanatory of the Greek arithmetic.

From this remaining fragment, it is reaſonably conjectured by Dr. WALLIS, that theſe two books related ſolely to that arithmetic; and thence he infers that the loſs of them is not greatly to be lamented. We learn from the fragment, indeed, that APOLLONIUS had compoſed a treatiſe on this ſubject, to which PAPPUS frequently refers; and Dr. WALLIS derives from it ſome information reſpecting the notation and algorithm of the ancient arithmetic. From the defective mode of notation among the Greeks, as compared with that of modern Europe, though there be much ingenuity in ſome of their methods, we need not be ſurpriſed at the great inferiority of their ſyſtem. Dr. Wallis remarks, that the buſineſs of the ſecond book of Pappus appears to be nearly equivalent to what is now conſidered as a very ſimple propoſition, viz. that the multiplication of any numbers, all or any of which have cyphers annexed, may be performed by multiplying theſe numbers without the cyphers, and then adding all the cyphers to the product. The firſt book he with much probability conjectures to have been employed about the ſimple operations of the addition and ſubtraction of numbers.* And I need only further remark, that there is no appearance among Dr. Simson's papers of his having directed his attention to this fragment.

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* See at the end of this Appendix a ſhort notice of the MSS. of PAPPUS, which are generally known to be preſerved in the libraries of Europe. It may be obſerved in this place, that ſome of them, beſides the Savilian MS., have the fragment of the ſecond book publiſhed by Dr. WALLIS.

The ſucceeding five books of Pappus are geometrical, and the laſt is almoſt entirely mechanical. They are all of a miſcellaneous character; though in ſome of them there is a certain degree of method, and titles have been prefixed to them, alluding to the principal matters contained in the ſeveral books.† Theſe titles do not appear to have been in the MS. followed by Commandine, as he does not mention them in his tranſlation; and probably, indeed, they did not belong to the original work, but were added by ſome commentator, perhaps of a much later age. Though I purpoſe to give a particular account of the contents of the ſeveral books, it may be proper firſt to mention the diſtinctions of geometrical propoſitions (eſpecially of problems) aſſumed by the ancients, as they are ſtated by PAPPUS in two paſſages of his work, expreſſed nearly in the ſame words.‡

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* See Walsii *Fragmentum Secundi Libri* Pappi, and the *Epilogus*.

† The MS. Boll. has thoſe titles, from which they are copied in this Appendix. The two Savilian MSS. at Oxford have them, and likewiſe the Pariſian MSS.

COMMANDINE's tranſlation of theſe paſſages is given in Appendix II. and the Greek not having been printed, is added from the two Savilian MSS.§ and the MS. BULL. for the ſatisfaction of thoſe who may wiſh to conſider them accurately. From theſe extracts it appears that it was the difficulty, or rather the impoſſibility, of reſolving ſome problems by the circle and ſtraight line, which ſuggeſted the inveſtigation of other curve lines, by the deſcription of which the ſolution of ſuch problems might be accompliſhed. The doubling of the cube,* and the triſection of an arch of a circle, were two celebrated problems, which exerciſed the ingenuity of the more ancient geometers, but which were found not to be reſolvable by plane geometry. From the very brief accounts which remain of theſe ſpeculations, it appears that the firſt attempt of producing new curves, which might be employed in geometrical ſcience, was from the ſection of a ſolid by a plane; and the only ſolids conſidered in the early ſtate of the ſcience, which, by ſuch a ſection could produce curves different from the circle, were the cylinder and cone. But as the ſections of the latter comprehended the curves reſulting from the ſections of the former, the three new curves, ariſing from the different poſſible ſections of the cone by a plane, obtained the name of Conic Sections. By theſe curves the two before-mentioned problems were eaſily reſolved; and from this origin, all problems requiring for their ſolution the deſcription of one or more of them, were called ſolid, though they had no other relation to ſolid figures.

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‡ The firſt paſſage is in the third book, fol. 4. b. in COMMANDINE's PAPPUS, (1588,) and the other in the fourth book, fol. 60.

§ Accurate copies of theſe paſſages from the two Savilian MSS. were procured for me, in the moſt obliging manner, by the Rev. Dr. ROBERTSON, Savilian Profeſſor of Geometry; who alſo gave me every facility for conſulting theſe MSS. and other books in the very curious collection placed under his care. I met with the ſame obliging attention, when conſulting ſome MSS. and books in the Bodleian Library.

* Uſually called the Deliacal Problem, from the well-known tradition about doubling the cubical altar at Delos.

Some other curves were alſo invented by ingenious men of thoſe times for the ſame purpoſe; but the Ancients did not purſue this branch of geometry, and conſidered only a ſmall number of ſuch lines, without having had any notion of the unbounded number which modern ſpeculations have brought into notice; and therefore, without propoſing any principle of ſyſtematic arrangement. All curves, beſides the Conic Sections, according to the account of PAPPUS, in the paſſages alluded to, were by the ancients denominated *Lines*, and problems reſolvable only by ſuch curves were called *Linear* Problems. All geometrical problems therefore, among them, were divided into three claſſes, *plane*, *ſolid*, and *linear*; and theorems were ſometimes diſtinguiſhed in the ſame manner, as they had reference to thoſe three claſſes of lines. The ſuperior lines treated of by PAPPUS, and other ancient writers, were the *conchoid*, the *ciſſoid*, the *ſpiral*, and the *quadratrix*; and a few others are ſlightly alluded to.

In the ſolution of the two celebrated problems, which are ſuppoſed to have given riſe to the inveſtigation of new curves, ſome of theſe laſt-mentioned curves have been employed, as well as the Conic Sections. And we are informed by PAPPUS, that the difficulty of deſcribing the Conic Sections with mechanical accuracy led ſome of the ancient geometers to employ thoſe higher curves, the deſcription of which was found to be more eaſy. The conchoid in particular was uſed for finding between two given ſtraight lines two mean proportionals, from which the doubling the cube was an obvious inference; and the triſection of an arch of the circle was accompliſhed alſo by the ſame curve, and likewiſe by the ſpiral and quadratrix.* From PAPPUS it appears, however, that the early Mathematicians had at firſt ſome reluctance in admitting either the Conic Sections or ſuperior curves in the ſolution of problems, conſidering them as not ſtrictly geometrical;† but afterwards theſe lines became objects of much curious inveſtigation, even among the ancients; and in modern times ultimately were of the moſt extenſive utility, both in abſtract and in phyſical ſcience.

The above-mentioned paſſages of PAPPUS naturally ſuggeſt an obſervation which may be ſtated in this place; that expreſſions occur on this ſubject, in the writings both of ancient and modern geometers, in which there ſeems to be ſome want of that preciſion and conſiſtency which properly belong to the language of mathematical enquiry. The deſcription of any geometrical line from the data by which it is defined, muſt always be aſſumed as poſſible, and is admitted as the legitimate means of a geometrical conſtruction: it is therefore properly regarded as a *postulate.* Thus the deſcription of a ſtraight line and of a circle are the poſtulates of plane geometry, aſſumed by EUCLID. The deſcription of the three *Conic Sections*, according to the definitions of them, muſt alſo be regarded as poſtulates; and though not formally ſtated like thoſe of EUCLID, are in truth admitted as ſuch by APOLLONIUS,[†] and all other writers on this branch of geometry The ſame principle muſt be extended to all ſuperior lines.

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* Theſe problems are reſolved in that manner by PAPPUS, in b. iii. and iv.

† Even ſome of the more early of the Mathematicians, reckoned modern, entertained a ſimilar notion; particularly VIETA.

It is true, however, that the properties of ſuch ſuperior lines may be treated of, and the deſcription of them may be aſſumed in the ſolution of problems, without an actual delineation of them. A plauſible though imperfect repreſentation of geometrical lines is indeed extremely uſeful in aſſiſting the imagination, when we are employed in the inveſtigation of their affections; but the degree of accuracy with which they are exhibited is of no importance to the truth of the reaſoning, or to the ſatisfaction with which it is perceived. For it muſt be obſerved, that no lines whatever, not even the ſtraight line or circle, can be truly repreſented to the ſenſes according to the ſtrict mathematical definitions;[‡] but this by no means affects the theoretical concluſions which are logically deduced from ſuch definitions. It is only when geometry is applied to practice, either in menſuration, or in the arts connected with geometrical principles, that accuracy of delineation becomes important.

[†] See Prop. 52, 53, 54, APOLLON. b. i. in which the deſcription of the three ſections is aſſumed preciſely according to the definitions given of them by APOLLONIUS.

[‡] CARTESII *Geometria*, lib. ii. at the beginning. It may alſo be remarked, that even in the *Elements* of EUCLID, particularly in the xith and xiith books, certain conſtructions are aſſumed, which, though perfectly conſiſtent with the rigour of geometrical demonſtration, would in mechanical practice be extremely difficult. But they muſt be conſidered as poſtulates, though not ſtated to be ſuch by EUCLID.

Among the ancients, the deſcription of curve lines with a certain degree of eaſe and accuracy was important; as we have reaſon to believe that ſuch deſcriptions were often uſed by them in practical applications of geometry to menſuration and mechanics. This the limited nature of their ſyſtem of computation rendered particularly expedient; and influenced by this conſideration, we may preſume, the Greek geometers, in the ſolution of problems reſolvable by the *Conic Sections*, ſometimes employed ſuperior curves, on account of the greater facility of deſcription; of which there are examples in the *Collections* of Pappus.* In recent times, however, the power of the modern analyſis, eſpecially with the aid of trigonometry, renders the accurate delineation of curves ſeldom neceſſary; and in the few caſes where it may be uſefully employed, thoſe curves ought no doubt to be aſſumed, the deſcription of which, in the exiſting ſtate of mechanical arts, moſt conveniently enſures the required degree of exactneſs.

But in a ſcientific conſideration of the ſubject, it is a principle admitted ultimately, both by the ancient and modern geometers, that the proper ſolutions of problems muſt be effected by the deſcription of the loweſt claſs of lines by which they are practicable; and that though conſtructions of problems by curves ſuperior to thoſe by which they are reſolvable, may be demonſtrably true, and in particular caſes, may be practically uſeful, yet in theory are to be rejected as irregular.

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* Pappus enumerates ſeveral ſolutions of the Deliacal problem, ſome of them purely mechanical, and one of them by the conchoid; a curve of a claſs ſuperior to the *Conic Sections.* Sir Isaac Newton even admits, that in certain caſes the conchoid may be preferred to the *Conic Sections,* on account of the ſimplicity of deſcription. *Arith. Universalis, Appendix, constructio Æquationum linearis.*

§. II. *The Third Book of* PAPPUS.

THIS book, as the title imports, contains geometrical problems, both plane and ſolid.*

The problems conſidered in this book are four: firſt, the duplication of the cube, or what is equivalent, the finding two mean proportionals between two given ſtraight lines, which is ſolid: ſecond, a problem reſpecting the mediatates: third, a ſort of paradoxical problem, to place two ſtraight lines, drawn from two points in one ſide of a triangle, to a point within it which may be greater than the other two ſides: fourth, to deſcribe the five regular ſolids in a ſphere. Theſe three are conſidered as plane.

The firſt problem, which was an object of much conſideration among the ancients, is reſolved by PAPPUS in ſeveral of the methods then known; but he begins with a refutation of an unſcientific attempt to reſolve it by plane geometry, which ſeems, in his time, to have excited ſome attention. The

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[*] The title in MS. BULL. is Πάππου Ἀλεξανδρέως συναγωγῶν† Γ, περιέχει δὲ προβλήματα γεωμετρικὰ ἐπίπεδα κάι στερεά.

[†] The transcript from the Parisian MS. in the Savilian library has συναγωγῶν μαθηματικῶν.

examination of it, which had been ſolicited from **Pappus** by the author and his friends, requires a long train of reaſoning, from the complex conſtruction which had been propoſed by this inaccurate geometer; and ſome things, aſſumed by **Pappus** in this argument, are demonſtrated in four propoſitions annexed to it. Both the text and the diagram have been injured by tranſcribers; but **Commandine**'s tranſlation is much amended by Dr. **Simson**, and a few of his corrections, indeed, are confirmed by the MS. **Bull.** and ſeveral in the figure are from the Pariſian MSS. communicated by Dr. **Moor**. Before giving the particular ſolutions of the Deliacal problem, **Pappus** premiſes a ſtatement of the ancient diſtinction of lines, already explained. In this account he mentions the ſolution of the Deliacal problem by means of the *Conic Sections*, without however giving a detail of that ſolution; but he explains ſeveral others, viz. three mechanical, by **Eratosthenes**, **Hero**, and **Pappus** himſelf; and a fourth of **Nicomedes**, by the deſcription of another curve, ſuppoſed to be invented by him, and called the *Conchoid*. **Eutocius**, in his commentary on **Archimedes** (Prop. 2. lib. ii. *de Sphæra et Cylindro)* gives ſeveral other ſolutions, (ten in all,) and particularly two by **Menechmus** by the interſection of two *Conic Sections*.—It may be obſerved alſo, that in ſome of the MSS. of **Pappus**, at the end of the third book are placed ſome variations of the mechanical ſolutions of this problem, not taken notice of in **Commandine**'s edition.†

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† In Prop. 24. lib. iv. The ſolution of this problem by the conchoid is repeated; and the ſolution in **Pappus**'s method in this book is alſo repeated in lib. viii. Prop. 11.

The ſecond problem reſpecting the *medietates* is branched out into twenty-one Propoſitions, with ſeveral definitions and explanations. He diſtinguiſhes three ſpecies, and when three lines (or three numbers) are either arithmetical, geometrical, or harmonical proportionals, according to the common definitions of theſe terms, they were anciently called *medietates:* while the term *analogy* was uſually applied only to the geometrical proportionals; ſo that ſtrictly an *analogy* is a *medietas*, but not converſely.

The principal problem[‡] which he propoſes, is to place the three medietates in a ſemicircle. The reſolution of this by PAPPUS is in Prop. 16th, but he begins with animadverting on a geometer of his time for his manner of treating the problem, though unjuſtly, as Dr. SIMSON obſerves in a note on this paſſage.[§]

Before this 16th Prop. however are ten problems according to COMMANDINE's enumeration, reſpecting the three medietates. But after that Propoſition he obſerves, that NICOMACHUS the Pythagorean, and others, had treated not only of theſe original medietates, but alſo of three[*] others which were added among the ancients, and four more among the moderns, making ten altogether, which he defines and illuſtrates by a number of Propoſitions to the 27th incluſive. Theſe, particularly the ancient medietates, were much conſidered in the Platonic School, in which probably they originated, and to which Pappus belonged. Eratosthenes compoſed two books on the medietates, (probably the *Loci ad medietates*) and though they were part of the τόϖος ἀναλυομένος, Pappus gives no deſcription of them, and has preſerved no Lemmata connected with them.[†] This may be conſidered as a preſumption of his opinion that in his time they were not regarded as important; and he further remarks of the more ancient medietates, that they were uſeful, chiefly in explaining alluſions to them in ancient writings; and probably this was the principal object of the details which he has given reſpecting them.

[‡] Called the ſecond. (The Deliacal problem being the firſt.) It is propoſed in fol. 7 b. COMMAND. PAPPI. And in the ſame page is the definition here referred to.

[§] Dr. SIMSON quotes VINCENTIO VIVIANI *de Locis solidis*, lib. iii. page 90, where there are ſome Propoſitions concerning the medietates, and alſo a remark of this miſtake of PAPPUS. I may add, that BLONDEL, in his *Resolution des 4 Problemes d'Architecture*, makes the ſame corrections of PAPPUS's cenſure; and adds, that the ſolution by PAPPUS (Prop. 16.) is defective. But his objection to that Prop. is rather verbal than geometrical. See BLONDEL, p. 36. 37.

[*] PROCLUS, in his *Commentary* on EUCLID, (p. 19. HERVAGII,) ſtates that EUDOXUS of Cnidus, about the time of PLATO, added three analogies (τρεις ἀναλογιας) to the three then known, and the former probably were the medietates alluded to in this place by PAPPUS.

The general nature of the third problem has been mentioned, and it is expanded into ſixteen Propoſitions. It is a ſort of paradox, and he refers to a treatiſe of Erycemus on Paradoxes, which, in the time of Pappus, was generally circulated, and probably in ſome eſtimation. Some of the Propoſitions are curious, but in the preſent ſtate of the ſcience they cannot be conſidered important, and are intereſting only as examples of the geometrical ſpeculations of former times, (Prop 28—43.)

[†] From the obſervation of Pappus at the end of his deſcription of the *Inclinations* of Apollonius, it would ſeem that theſe *Loci ad medietates* were *plane*, and, in the order of the τόϖος ἀναλυομένος, placed laſt.

The fourth and laſt general problem of this book is to inſcribe in a ſphere the five regular ſolids; and as theſe ſolids were objects of much ſpeculation in the Platonic ſchool, they were naturally attended to by **Pappus**, who belonged to it. This problem occupies the remainder of this book in the laſt ſixteen Propoſitions, of which eleven are preliminary to the other five; and theſe laſt contain the ſolution of the problem reſpecting the before-mentioned five regular ſolids.

§. III. *Of the Fourth Book.*

The fourth book is miſcellaneous, containing theorems of the three ancient claſſes of Propoſitions formerly mentioned; plane, ſolid, and linear.* The firſt may be remarked as an extenſion of the 47. 1 Elem.; and the 4th is a curious theorem, with an analyſis and compoſition, which was rendered more general by Dr. M. **Stewart**, in a paper publiſhed in the firſt volume of the Edinburgh *Phyſical Eſſays*;§ and ſome Propoſitions alſo are added by Dr. **Simson**, in his notes.

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* The title in MS. **Bull.** is as follows: Πάππου συναγωγῆς δ' ὅπερ ἐστιν ἀθηρῶν‡ θεωρη“μάτων ἐπιπέδων”, καὶ στερεῶν καὶ γραμμικῶν."

‡ Probably it ought to be ἀνθηρῶν, as it is in the title of the eighth book.

§ In Dr. **Stewart**'s Paper, in that volume, are ſeveral Propoſitions connected with this propoſition of **Pappus**, and which are analyſed and demonſtrated in the ancient manner. A ſcholium, containing ſome conic theorems without demonſtrations, is added; and at the end of it is a theorem, alſo without demonſtration, which is truly a Poriſm, and is analyſed and demonſtrated by Dr. **Simson**, Prop. 91 of his treatiſe. Dr. **Stewart**, from delicacy no doubt to Dr. **Simson**, who wiſhed to keep his expo-

The 10th is the caſe of one of the problems of the *Tactions* of APOLLONIUS, in which three circles touching each other are given, to which the three preceding Propoſitions are preliminary; all however requiring the corrections remarked by Dr. SIMSON.†

Before the 13th Prop. a property of the figure called *Arbelon*‡ is ſtated, as an ancient Propoſition generally known, and of which the demonſtration is in the 16th, the intermediate Propoſitions being neceſſary to it.

At the 19th Prop. he gives the definition of the Spiral of CONON, and demonſtrates ſome of its principal properties.‖ This is the curve fully treated of by ARCHIMEDES, and in a ſubſequent part of the book it is employed in the ſolution of ſome problems. PAPPUS (Prop. 24) repeats the definition of the

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ſition of the Poriſms ſecret till he ſhould publiſh his treatiſe, ſtates the Propoſition as a theorem, though he was well acquainted with Dr. SIMSON's diſcovery, and had inveſtigated many curious Poriſms.——See *Edinburgh Phyſical Essays*, vol. i. p. 141—172.

† There are ſome amendments on the 12th, neceſſary to make the demonſtration complete.

‡ The Arbelon (ἄϱβηλον) is mentioned in the *Lemmata* of ARCHIMEDES, (Prop. 4, 5, 6;) and it is remarkable, that though PAPPUS in his *Collections*, and particularly in this book, often quotes ARCHIMEDES, there is no alluſion to the *Lemmata* in his long and curious diſcuſſion of the properties of the Arbelon. There is alſo a lemma of COMMANDINE's, for demonſtrating a Propoſition of PAPPUS on this ſubject, (Prop. 14,) which is the firſt Propoſition of the *Lemmata* of ARCHIMEDES, and which COMMANDINE afterwards (fol. 52, b.) aſſerts to be compoſed by himſelf. From this it may be inferred, that COMMANDINE certainly had never ſeen the *Lemmata* of ARCHIMEDES, and moſt probably neither had PAPPUS. The *Lemmata* have never been found in Greek, and have by ſeveral learned men been ſuppoſed not to be the work of ARCHIMEDES; and the circumſtance now mentioned favours that ſuppoſition.

‖ In Prop. 21, reſpecting the Spiral, the demonſtration is conducted on principles like the method of Indiviſibles of CAVALLERIUS.

conchoid of Nicomedes, and the ſolution of the Deliacal Problem by it, as in the preceding book: he afterwards employs this curve for reſolving another celebrated problem of antiquity, the triſection of an arch of a circle, which was ſound impracticable by plane geometry; and which, with the Deliacal Problem, rouſed the efforts of mathematicians to inveſtigate new curves for reſolving them.

He proceeds to explain the origin and properties of the quadratrix, (*τετραγωνίζȣσα*,) aſſumed by Denostratus and Nicomedes, ſubſequent geometers, for the quadrature of the circle, from which it obtained its name. But he obſerves that another geometer, Sporus, was not ſatisfied with this application of that curve, and he gives ſome detail of the objections propoſed by Sporus; and particularly, that in the conſtruction of this line the problem to be reſolved by it is in ſome meaſure aſſumed. Pappus however, beſides the common deſcription of its origin and of its properties, gives what he conſiders as a more ſtrictly geometrical deſcription, by *Loci ad Superficiem.**

After ſeveral Propoſitions on this ſubject, there is a repetition of the ancient claſſification of lines, and nearly in the ſame words† in which it was ſtated in the third book. It ſeems to be introduced here for illuſtrating the problem of the triſection of an arch of a circle, as in the former book it had a more particular reference to the Deliacal problem.

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* This is Prop. 28, which is ſo much corrupted in Commandine as to be ſcarcely intelligible; and in Note G. at the end of the preceding Memoir is placed Dr. Simson's corrected edition of it. Another deſcription of it is given in Prop. 29.

† The Greek of this paſſage is added in Appendix II.

This problem is reſolved by **Pappus** in various ways; firſt, by a ſolid inclination, to which Dr. **Simson** adds an improvement; then by an hyperbola without the inclination;* and he mentions that in another treatiſe† he had reſolved it by means of the conchoid, of which the method is obvious.

He ſubjoins ſome linear problems, ſuch as dividing an angle or arch of a circle in a given ratio; the conſtruction of an iſoſceles triangle, of which each of the angles at the baſe ſhall have a given ratio to the angle at the vertex; which are eaſily reſolved, by aſſuming the quadratrix or ſpiral, as proper means of geometrical ſolutions.

Beſides the very important emendation of Prop. 28th and 29th of this book by Dr. **Simson**, there are various other neceſſary corrections in other Propoſitions, and ſome uſeful readings adopted by him from MS. **Bull.** which it is unneceſſary to ſpecify. At the end of this book, there is in MS. **Bull.** a very imperfect ſketch of a problem ſaid to have been uſed by **Archimedes**, in his attempt to find a ſtraight line equal to the periphery of a circle; and alſo that this uſe of it had been animadverted on. It is not in **Commandine**'s tranſlation, and in its preſent ſtate is altogether unintelligible.§

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* This Inclination is alluded to in Note F. p. 99.

† From the very brief notice of this work, it ſeems to have been a comment on the Analemma of **Diodorus**: "*και ἡμεις ἐν τῷ ἔις τὸ ἀναλήμμα Διοδωρȣ τρίχα τεμεῖν τὴν* "*γωνίαν βȣλȣμένοι κεχρήμεθα τῇ προειρημένῃ γράμμῃ.*" MS. **Bull.** Vide **Command. Papp.** fol. 56, a. l. 6.

§ This Propoſition, in the ſame unintelligible ſtate, is found in Savil. MS. No. 3, and at the end of the fourth book.

§. IV. *The Fifth Book.*

To the fifth Book there is a preface, in which **Pappus** makes ſome philoſophical obſervations on the curious inſtincts of animals, which in many caſes ſupply the place of reaſon. He mentions more particularly the inſtinct of bees, by which they conſtruct the receptacle for their proviſion on geometrical principles, by employing the hexagon, which of the three equilateral and equiangular figures that occupy the ſpace round a point, ſupplies, with the ſmalleſt labour, the moſt convenient accommodation. This obſervation ſeems to be introductory to the purpoſe of the book, which, at the end of the preface, he ſtates to be; to prove that of plane, figures which are equilateral and equiangular, and have equal perimeters, the greater ſpace is contained by the figure with the greater number of ſides; and that of all plane figures, of equal perimeters, the circle is the greateſt.*

This ſubject of iſoperimetrical figures is largely treated of in this book, containing fifty-ſeven Propoſitions.‡ Beſides

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* The title of the fifth book in MS. Bull. is the following, Πάππȣ Ἀλ ξανδρέως συναγωγῆς ε̄, περιέχει δὲ συγκρισεῖς τῶν ἴσην περίμετρον ἐχόντων ἐπιπέδων σχημάτων πρὸς ἀλλήλα τε καὶ τὸν κύκλον,|| καὶ συγκρισεῖς τῶν ἴσην ἐπιφανειαν ἐχόντων στερεῶν σχημάτων πρὸς ἀλληλα καὶ τὴν σφαῖραν.

|| Note in MS. Bull. it is erroneously τῶν κυκλῶν, but in the Savil. MS. No. 3. it is τὸν κύκλον.

‡ In Prop. 1. an eaſy Propoſition is aſſumed, which is demonſtrated in a Lemma (Note B. on that Prop.) by Commandine. It is remarkable, that a Propoſition of the ſame import in Greek is found on the margin of MS. Bull. And in the text of MS. Savil. No. 3, is a ſimilar Propoſition. It would require however the examination of other MSS. to aſcertain the hiſtory of this Lemma.

the object mentioned in the preface, ſeveral other things are conſidered. It is proved, that of plane figures with equal perimeters, the greateſt is that which is equilateral and equiangular. This principle is extended to ſolids; the regular ſolids (as they are called) are compared, and it is proved, that, of thoſe with equal ſurfaces, the greateſt is that with the greater number of ſides. Theſe Propoſitions are meant no doubt to be introductory to a demonſtration of the Propoſition, that of all ſolids with equal ſurfaces the greateſt is the ſphere. There are many references to ARCHIMEDES, and ſeveral of the Propoſitions in his treatiſe of the Sphere and Cylinder are demonſtrated in this book by PAPPUS,* and, as he mentions, in a different manner.

It was formerly obſerved, that a number of Propoſitions in it, reſpecting the compariſon of the five regular bodies having equal ſurfaces, were demonſtrated only ſynthetically from the brevity and the facility of communication in that method; and that this explains the omiſſion of the analyſis in many ancient Propoſitions, though there can be no doubt of that analyſis having been uſed by the authors in the inveſtigation of them. He introduces the Propoſitions reſpecting theſe regular ſolids by a general diſſertation on them before Prop. 18. in which he profeſſes great reverence for the doctrines of the divine PLATO, in whoſe ſchool it is ſuppoſed that theſe bodies were firſt defined, and their relations fully treated of. He remarks, that it is aſſumed by philoſophers, without proof, that the ſphere is the greateſt of ſolids, having equal ſuperficies: he proceeds to give ſome popular illuſtrations of that truth; and in Prop. 18. he propoſes a compariſon of the ſphere with the regular ſolids. In the end of the book he ſtates, what is eaſily aſcertained, that there can be only five regular ſolids, which are comprehended by equal, ſimilar, and equilateral polygons, viz. the tetraedron, cube, octaedron, dodecaedron, and icoſaedron; in ſome of the Propoſitions Dr. SIMSON has remarked ſeveral proper and neceſſary corrections.

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* In Prop. 35. (fol. 96. COMMAND.) is a very material improvement of the demonſtration by Dr. SIMSON, by which the long note (Z) of COMMANDINE becomes unneceſſary. This may be remarked in many of the Doctor's corrections, which, even when ſhort, often ſuperſede the uſe of long and ſometimes tedious comments by COMMANDINE.

§. V. *Sixth Book.*

THE ſixth Book of PAPPUS, as the title intimates,‡ is employed chiefly in explaining and correcting ſome Propoſitions of THEODOSIUS, and ſome other ancient writers, in treatiſes containing chiefly, what is popularly called, the doctrine of the ſphere. In a ſhort preface the object of this book is ſtated, with a reference to three examples of Propoſitions criticiſed in it, viz. the 6th, iii. THEODOSIUS *on the Sphere*; the ſecond Prop. of EUCLID's *Phænomena*; and the fourth Prop. of THEODOSIUS *on Days and Nights*. This collection of treatiſes, obtained, in the Alexandrian ſchool, the name of μικρὸς ἀστρονόμος, or, as it is in PAPPUS μικρὸς ἀστρονομούμενος. Among the Arabians they were called *Libri intermedii inter* στοιχοιωτὴν *et magnum conſtructorem*, that is, between EUCLID and PTOLEMY:

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§ The title of this book in MS. BULL. is Πάππου Ἀλεξανδρέως συναγωγῆς ἕκτον, περιέχει δὲ τῶν ἐν τῷ μικρῷ ἀστρονομουμένῳ θεωρημάτων ἀπορῶν λύσεις.

and in contraſt to the greater work of the latter (the Almageſt, ſometimes called *μέγας ἀϛρονόμος*) theſe ſmaller tracts got the name of *μικρὸς ἀϛρονόμος*.

According to Vossius, this collection contained the nine following works, viz. Theodosii *Sphærica*, Euclidis *Optica*, ejuſdem *Phænomena*, Theodosii *de Habitationibus*, ejuſdem *de Noctibus et Diebus*, Autolycus *de Sphæra*, ejuſdem *de Ortu et Occaſu*, Aristarchi *de magnitudinibus et distantiis Solis et Lunæ*, Hypsiclis *ἀναφορικὸν*, ſive *de ascensionibus*.† Fabricius, when ſtating this collection, includes in it alſo Menelai *Sphærica*, and Euclidis *Data* et *Catoptrica* :‡ and he adds, that theſe treatiſes, or moſt of them, are often found together in Greek MS. in the libraries of Italy and France.

It may be alſo obſerved, that all theſe treatiſes, except that of Hypsicles, are quoted or alluded to in this ſixth book, in which Pappus examines ſeveral ἔνϛασεις *(Instantiæ)*, to be found in them. By this term he appears to underſtand the objections to certain Propoſitions in theſe writers, or the limitations and exceptions neceſſary to be made reſpecting them, but which had been omitted by the reſpective authors. Proclus gives an explanation of this term;§ and from his diffuſe and not very ſatisfactory account, this word ἔνϛασις *(Instantia)* ſeems to have been applied to any objection made to a Propoſition, or even to the denial of it, as preparatory to an indirect demonſtration of it, by proving the abſurdity of that denial. In **Pappus**, however, the application ſeems to be confined to the former meaning, with which the examples which he gives, from **Theodosius** and others, correſpond.[†]

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† Ger. Vossius *de Scientiis Mathematicis, et Chronologia Mathematicorum*, cap. xxxiii. §. 18.

‡ Fabricii *Biblioth. Græc.* Harles, Hamb. 1795. tom. iv. p. 16, and alſo p. 212. Savilius in Prælect. II. in Euclidem.

§ Proclus, p. 58. Hervagii; p. 121. Barocii.

Prefixed to Prop. 39 of this book, is a long diſcuſſion concerning the poſitions in the before mentioned tract of **Aristarchus Samius** and this, with the 39th, 40th, and 41ſt Propoſitions on the ſame ſubject, is inſerted by Dr. **Wallis** in his edition of *Aristarchus* publiſhed at Oxford in 1688. He adds the Greek of this extract from **Pappus** according to the two Savilian manuſcripts, which he takes an opportunity of characterizing.[*]

In the explanation and correction of ſeveral paſſages in theſe tracts by **Pappus** are introduced ſome curious Propoſitions, by which the doctrine of the ſphere of that period was improved; and there are ſome Propoſitions relating to a ſort of perſpective or projection of the ſphere, which are connected with Propoſitions of **Euclid**'s *Phænomena.* It may be alſo obſerved, that in four Propoſitions, viz. 31, 32, 33, 34, which are preliminary to ſome following Propoſitions reſpecting the ſphere, ſome diſtinctions of magnitudes are ſtated as examples, which may be either increaſed or diminiſhed without limit; or may be increaſed indefinitely, while there is a limit to the decreaſe, and converſely. And I mention them here, merely to remark the inaccuracy of **Commandine**'s tranſlation, by introducing the term *infinite* without authority from the original. Dr. **Simson** obſerves that he had not conſidered this book ſo particularly as he purpoſed afterwards to do, but which he appears not to have accompliſhed: at the ſame time he had made ſome neceſſary and uſeful corrections in ſeveral Propoſitions as they ſtand in **Commandine**'s tranſlation.*

[†] Examples of *instantiæ* may be found in Prop. 21, of this ſixth book, to which the preceding ten Propoſitions refer. Alſo in Prop. 51, (fol. 145. a. at the bottom, **Command**.) Other criticiſms on **Euclid**'s *Phænomena*, and alſo on **Hipparchus**, are mentioned in ſubſequent Propoſitions of this book. There are alſo many references to **Ptolemy**.

[*] In his *præfatio ad Lectorem* he obſerves, "quorum (ſc. MSStorum) qui "elegantius ſcribitur (ſc. No. 3.) eſt mendoſior; quique feſtinantius et minus "eleganter ſcribitur (ſcil. No. 9.) eſt emendatior."

§. VI. *Seventh Book.*

This Book, as the title imports, is wholly employed on that curious ſubject, the ancient analyſis.†

The collection of treatiſes known by the name of *τόπος ἀναλυομένος*, next to the *Elements* and *Data* of **Euclid**, were important for facilitating the reſolution of geometrical problems. They were compoſed, by the elder **Aristæus**, **Euclid**, and **Apollonius**;‡ but at what time the collection obtained the name by which they are diſtinguiſhed in the preface to this book, is unknown. They muſt have been compoſed at different periods, from the known interval of the times of the reſpective authors, who all lived above 500 years before the age of PAPPUS. The expreſſion of PAPPUS is not definite; yet it ſeems to imply that the collection was made, and the name impoſed, before his own time, for his expreſſion ſeems to ſuppoſe the order of the books as exiſting.‡

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* It may be obſerved, that in the MS. **Bull.** and in the Savil. MS. No. 3, are ſome marginal notices and at the end of Prop. 59, a ſcheme reſpecting climates which may deſerve conſideration; in that Propoſition alſo are references to **Ptolemy**.

† The title of the 7th book, from one of the Pariſian MSS. No. 2368, is as follows; *Πάϖϖȣ 'Αλεξανδρεῶς συναγωγῆς ζ ὃ περιέχει τὴν τάξιν, καὶ τὴν ϖεριοχὴν καὶ τα λήμματα τȣ̃ ἀναλυομένȣ τόϖȣ.*

‡ In this firſt ſtatement of the authors in the preface to this viith book theſe three only are mentioned. In a ſubſequent more particular account (alſo in this preface) **Pappus** mentions two books on the *Medietates* by **Eratosthenes**. But neither this work of **Eratosthenes**, nor the *Loci* of **Aristæus**, nor the *Loci ad Superficiem* by **Euclid**, are deſcribed by **Pappus**.

The preface to this book, one of the moſt valuable remnants of ancient geometry, contains a particular deſcription of the nature and contents of a certain number of theſe treatiſes, which, both from the ſubjects of them, and from their being thus diſtinguiſhed by PAPPUS, may be conſidered as by far the moſt important part of the collection; and the book itſelf contains a number of Lemmata or elementary Propoſitions aſſumed (probably without proof) in the treatiſes deſcribed in the preface, but of which the demonſtrations are added by PAPPUS.

PAPPUS, however, premiſes a ſhort but accurate account of the ancient method of analyſis and ſyntheſis; of which a free tranſlation will be more ſatisfactory than any abridgement, for conveying a correct notion of this curious branch of ſcience, ſo much valued among the ancients, and, till the time of Dr. HALLEY and Dr. SIMSON, ſo little underſtood among the moderns.

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‡ Dr. HALLEY, whoſe authority in ſuch points is very great, at the end of his preface to the *Sectio Rationis*, intimates an opinion that the collection was made by PAPPUS. But the circumſtances now mentioned render this inference from the words of PAPPUS at leaſt doubtful; and Dr. SIMSON has remarked another miſtake of Dr. HALLEY's in the before mentioned preface reſpecting the principal object in forming this collection.

It is employed both for the reſolution of problems, and alſo for inveſtigating the truth of theorems either aſſerted or conjectured to be true; though it is in the former claſs of Propoſitions that the uſe and importance of this method is chiefly known.

In the analyſis of a problem, what is propoſed to be done, or to be found, is ſuppoſed to be obtained; and the conſequences of ſuch an aſſumption are ſucceſſively deduced by means of all previouſly known practical Propoſitions, which appear to be connected with it, till at length we arrive at ſome concluſion, which, from the ſtate of geometrical ſcience at the time, we know to be practicable; and thence the ſolution of the problem is deduced Syntheſis, or compoſition is the reverſe of analyſis; and by aſſuming the laſt practicable conſequence of the analyſis, we proceed in a contrary order through the ſeveral ſteps of that analyſis till we neceſſarily reach the conſtruction of the thing required, and then the problem is reſolved. In this analyſis however, if we arrive at a conſequence which we know, from previous Propoſitions, to be impoſſible, then the problem propoſed muſt itſelf be impoſſible; and further, if we find, from the progreſs of the compoſition, that in certain relations of the given magnitudes the conſtruction is practicable, while in others it becomes impoſſible; the aſcertainment of theſe relations becomes a neceſſary part of the ſolution, and is called the determination of the problem.

In the analyſis of a theorem,[*] the aſſertion in the enunciation is aſſumed as true; and by reaſoning from it, by the application of known geometrical Propoſitions which appear to be connected with it, we trace the ſucceſſive conſequences of the firſt aſſumption, till we arrive at ſome one which we know to be true, or to be falſe. If we arrive in this manner at a true Propoſition, by proceeding from it, in a contrary order, through the ſeveral ſteps of the analyſis, we ſhall neceſſarily arrive at the propoſed aſſertion; and this laſt proceeding will be a demonſtration or compoſition of the theorem. In like manner, if in this inveſtigation we arrive at a concluſion which we know to be falſe, from the ſame neceſſary concatenation of Propoſitions, we infer the falſehood of the aſſumed aſſertion; and if it were requiſite, we might alſo demonſtrate that inference. But it not being my deſign, in this general account of the ſeventh book of Pappus, to attempt a full expoſition of the ancient analyſis, which, for the uſe of thoſe who have not conſidered the ſubject, would require alſo the illuſtration of examples, I can only refer to thoſe treatiſes where ſuch explanatory examples of the practice of analyſis may be found.[†] I ſhall therefore only further obſerve, that, in this practice, ſome of the principles and rules commonly laid down by writers on the application of the algebraical analyſis to the ſolution of geometrical problems, may here alſo be uſefully employed; though ſeveral of them, no doubt, are more peculiarly fitted for the modern ſyſtem.[†]

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[*] The analyſis of theorems is often uſeful. A theorem may be propoſed to a geometer, that he may inveſtigate the demonſtration; it may be only ſuſpected to be true from analogy, from the apparent relations of magnitudes in geometrical diagrams, and even from accident; but in all ſuch caſes an analyſis will aſcertain the truth or falſehood of the aſſumed Propoſition.

[†] In the remaining works of Apollonius, as publiſhed by Dr. Halley, and alſo in Pappus, many problems and theorems are treated analytically. In Dr. Simson's different works are various examples in the pure ſtile of the ancient geometry. The analyſis of theorems is well illuſtrated in Dr. Stewart's *Propositiones Geometricæ more veterum demonstratæ*. For problems, I may refer to Dr. Horsley's (Biſhop of St. Aſaph) *Delectus Problematum*; and to Mr. Profeſſor Leslie's *Geometrical Analysis* in his *Elements of Geometry*. In that treatiſe the general problems of Apollonius, mentioned in this book of Pappus, are introduced as examples.

After this general account of the ancient analyſis, **Pappus**, in his Preface, proceeds to enumerate the books contained in the *τόϖος ἀναλυομένος*, and afterwards to give a particular account of the nature and contents of a certain number of them, viz. of twenty-four; the whole number being thirty-three.[‡]

Theſe twenty-four books are, I. *Data* of **Euclid**; III books *De Poriſmatis*, alſo by **Euclid**; and the following twenty by **Apollonius**: II. *De Sectione Rationis*; II. *De Sectione Spatii*; II. *De Sectione Determinata*; II. *De Tactionibus*; II. *De Inclinationibus*; II. *De Locis Planis*; and VIII. *De Conicis.*

Theſe books, though all uſeful in facilitating the ſolution of geometrical problems, yet are of different characters, and promote the object of the whole collection in different ways; as may appear from the many references to them in this Memoir, of which the following ſhort recapitulation is added.

[†] See **Newt.** *Arith. Univerſalis*, cap. 23.——See alſo ſome obſervations of Dr. **Simson** on the ancient analyſis, in Note K. at the end of the Memoir, p. 121—128.

[‡] In this preface **Pappus** mentions his purpoſe of giving a deſcription of theſe books as far as the *Conics* of **Apollonius**; and therefore, according to his arrangement, which is not in the order of time, he appears to omit by deſign the nine following books: V. of **Aristæus** *de Locis Solidis*, (which were no doubt ſuperſeded by the *Conics* of **Apollonius**;) II. of **Eratosthenes** *de Medietatibus*; and II. by **Euclid**, *de Locis ad Superficiem.*

I. *The Data of* Euclid. This book, one of the few which have been preſerved in the original language, may be conſidered as a collection of elementary problems; and the demonſtrations as given by Euclid, would become the analyſes of theſe Propoſitions, had they been enunciated as problems. The book was conſtantly uſed by the ancients in their reſolutions of problems, though it was not their practice to quote the particular Propoſitions; and this application of the Data was continued among ſuch of the moderns as follow ſtrictly the ancient method of analyſis, and is now ſo well known, that it is unneceſſary to give any detail of it.* The *Sectio Rationis*, *Sectio Spatii*, *Sectio Determinata*, and the Treatiſes *De Tactionibus*, and *De Inclinationibus*, are all general problems of frequent recurrence in geometrical inveſtigations, and were reſolved by Apollonius in the moſt complete manner, all the poſſible caſes being diſtinguiſhed; and of each caſe a ſeparate analyſis and compoſition are given, with the reſpective determinations in all thoſe caſes which required them.†

The uſe of theſe general problems, as has been repeatedly mentioned, was for the more immediate reſolution of any propoſed geometrical problems which could be eaſily reduced to a particular caſe of any one of them.‡ By ſuch a reduction the propoſed problem was conſidered as fully reſolved; becauſe it was then neceſſary only to apply the analyſis, compoſition, and determination of that caſe of the general problem to this particular Propoſition, which was ſhewn to be comprehended in it. The apparatus of ſeparate ſolutions, with the determinations of every poſſible caſe which is eſſential to the uſe of theſe general problems, may appear forbidding; and if regularly peruſed without examples of the application, may ſometimes appear tedious and unintereſting; and this perhaps, may have created ſome prejudice againſt the ſtudy of them. Dr. Halley, indeed, ſeems to think that the books of the *τόπος ἀναλυομένος*, were intended merely for the inſtruction of beginners in the ſtudy of the geometrical analyſis:§ but it is juſtly obſerved by Dr. Simson, that though they may be moſt advantageouſly employed for that purpoſe, yet that it is manifeſt from the contents of thoſe books, of which he gives ſome detail, and alſo from the manner in which they are referred to in the ſmall remains which we poſſeſs of the ancient geometry, the chief object of them was what has now been ſtated. Even in Pappus there are ſome examples of problems being reſolved by a reduction of them to caſes of theſe general problems. In the 85th Prop. of this ſeventh book, a problem, uſeful in the *Treatiſe of Inclinations*, is by analyſis brought to a caſe of the *Sectio Determinata*, as was formerly mentioned in the Memoir. Another example is Prop. 164, of the ſame book, the laſt lemma belonging to the Poriſms; which is a problem reſolved by Pappus, by reducing it to a caſe of the *Sectio Spatii*, and the compoſition of the problem is conſidered as compleated, merely by a reference to the conſtruction of that caſe. This tract of Apollonius being loſt, and no reſtoration of it having been made in the time of Commandine, the caſe referred to is reſolved by him, though in a tedious manner, which may be compared with the ſolution of it in Dr. Halley's reſtoration of that work.*

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* See in Note K. at the end of the Memoir, ſome remarks on the Data, in the correſpondence between Dr. Simson and Mr. Scott.

† Pappus, in this preface, alluding to a problem connected with the Treatiſe of Tactions, ſtates with brevity, but with preciſion, what is requiſite in a perfect ſolution of ſuch general problems: "καὶ ταῦτα ἀναλῦσαι, καὶ συνθεῖναι, καὶ διορίζεσθαι κατὰ ϖτῶσιν."

‡ When a problem can be reſolved with equal facility by the uſe of known elementary Propoſitions, a reference in ſuch a caſe to any of theſe general problems becomes unneceſſary.

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§ Halleii *Sect. Rationis*, Præfat. See alſo *Loci Plani*, Simson's Præfat. p. vii.

The other books, deſcribed in this preface by Pappus, are uſeful for the ſame purpoſe of facilitating the ſolution of geometrical problems, though in a manner different from what has juſt been mentioned of the preceding treatiſes. The treatiſes of *Loci* in particular are very important, and in a method well underſtood by thoſe even ſlightly acquainted with either the ancient or modern geometry. The *Loci Plani* of Apollonius are uſeful for the ſolution of plane problems; and often alſo, with *Loci* of a higher order, may be required in the ſolution of ſuperior problems. Though the original work of Apollonius be loſt, the elegant and compleat reſtoration of it by Dr. Simson leaves nothing to be regretted on that ſubject. The *Loci Solidi* of Aristæus might have been very ſerviceable in the ſolution of ſolid problems, but it has periſhed; and probably, even in the time of Pappus, had been ſuperſeded by the great work of Apollonius on *Conic Sections*, from which not leſs important aid might have been derived for reſolving that claſs of problems.†

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* Schooten alſo reſolves this problem, *Exercit. Mathem.* p. 104, 105, and ſeems to blame Pappus for not reſolving it directly, but by a reference to a caſe of the *Sectio Spatii*; from which it appears, that Schooten was not appriſed of the true uſe of theſe ancient books. His Propoſition alſo is not ſo general as it is ſtated in Pappus. See Dr. Simson's edition of this Prop. *Opera Reliqua*, p. 529.

† As ſeven books of the *Conics* of Apollonius have been recovered, and the eighth reſtored in a ſatisfactory manner by Dr. Halley, it is unneceſſary to enter

The poriſms of Euclid were a peculiar claſs of Propoſitions, and uſed among the ancients in the ſolution of ſome of the moſt difficult geometrical problems. The long account of them in this preface has been particularly unfortunate in ſuffering from the injuries of time; ſo that till Dr. Simson's perſevering induſtry and ingenuity were employed on them, no ſatisfactory explanation of the nature of theſe Propoſitions could be diſcovered, nor could any ſingle Propoſition of Euclid's treatiſe be reſtored. But in the preceding Memoir ſo full an account of Dr. Simson's reſtoration of them has been given, that it is unneceſſary in this place to make any further obſervations reſpecting them.

With reſpect to the τόπος ἀναλυόμενος, I ſhall only further obſerve, that on the revival of mathematical learning in Europe, if the ſtudy and application of the ancient analyſis had continued, without being nearly ſuperſeded by the uſe of the modern algebra; it is highly probable that later geometricians would, from time to time, have made additions to the ancient collection; and would have inveſtigated various other general problems, with compleat ſolutions of their ſeveral caſes, for the ſame general purpoſe for which theſe ancient books were compoſed by Euclid and Apollonius.

At the end of the deſcription of the *Conics* of Apollonius, in this preface to the ſeventh book of Pappus, there are ſome intereſting obſervations of a more general nature, which have

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into any diſcuſſion reſpecting the account of this work by Pappus. It is, however, both curious and intereſting; and Dr. Halley, in his preface to the corrected edition of this preface to the ſeventh book of Pappus, makes ſome valuable obſervations on it. Dr. Simson adds ſome uſeful notes on the *Lemmata* of this work.

been particularly remarked by Dr. HALLEY. I allude to the account of the celebrated ancient problem of the "*Locus ad* "*tres et quatuor rectas*;" and the extenſion of it mentioned by PAPPUS to the caſes where there are a greater number of ſtraight lines than four, to which lines are drawn in given angles from the point, of which the *Locus* is to be inveſtigated. The original *Locus* is ſolid, but theſe other caſes produce *linear Loci*, that is, curves of ſuperior orders; which, however, had not been inveſtigated in the time of PAPPUS. In ſome general obſervations on the ſubject by PAPPUS, it may be remarked that he mentions the method of expreſſing dimenſions above the cubic, by means of compound ratios.

This laſt article of the preface is concluded by a diſtinct enunciation of the celebrated theorem of GULDINUS, which, twelve centuries after the age of PAPPUS, excited much curioſity and admiration. Whether this theorem was invented by PAPPUS, or by ſome other geometer, is not ſtated. It is proper to remark, however, that no imputation is conveyed on the originality of the diſcovery by GULDINUS; as in COMMANDINE's tranſlation, which was the only account then in print, the Propoſition is altogether unintelligible.* But as this portion of the preface has not been conſidered in Dr. SIMSON's notes, it becomes unneceſſary in this place to enter into any further diſcuſſion of it.

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* The improved tranſlation of this preface of PAPPUS, by Dr. HALLEY, appeared only in 1706, long after the time of GULDINUS; and in it this theorem from PAPPUS was firſt intelligibly publiſhed. MONTUCLA indeed remarks, (tom. i. p. 325,) that the work of GULDINUS was publiſhed before the ſecond edition of COMMANDINE's PAPPUS in 1660. But that edition could not have given him any aid, for it contained no valuable improvement of the former; and in this particular point, it is an exact tranſcript of the paſſage in the firſt edition.

Having given ſo particular an account of the preface of the ſeventh book, (certainly one of the moſt curious remains of ancient geometry,) a very ſhort ſtatement of the contents of the book itſelf will be ſufficient. It conſiſts of a great number of Propoſitions, (238,) ſome of which are problems, but the greater part theorems. They are called *Lemmata*, and divided into claſſes according to the ſeveral treatiſes deſcribed in the preface, to which they reſpectively belong. They appear in general to be elementary Propoſitions aſſumed, but we may ſuppoſe not demonſtrated, in theſe ancient books, from which they were collected probably by PAPPUS; and the collector has added the demonſtrations. Some of them indeed are curious and important Propoſitions; but the greater number of them are chiefly valuable for the aid which they give in the attempts of modern geometers in reſtoring accurately the loſt books of the τόϖος ἀναλυομενος. In ſuch attempts, it was an intereſting object, not merely to reſolve the general problems contained in that collection, with their caſes and determinations, but alſo, where it was poſſible, to follow the particular mode of ſolution employed by the original authors. Theſe Lemmata[‡] preſerved by PAPPUS have ſupplied the means of doing this; and when proper analyſes and compoſitions were diſcovered, which required alſo the uſe of the particular Lemmata mentioned by Pappus, there was a high degree of probability, that ſuch ſolutions were truly thoſe given by Euclid and Apollonius.

[‡] According to PROCLUS, in his comment on Prop. 1. I. Elem. a Lemma (ſumptio BAROC.) is an aſſumption of a geometrical truth, in the demonſtration of a Propoſition; which truth, however, has not been demonſtrated by known geometrical writers. In this ſenſe is the term uſed by PAPPUS, and the Lemmata of the ſeventh book ſeem to be of that deſcription. It appears from the books of APOLLONIUS's *Conics* ſtill preſerved, that the Lemmata of PAPPUS belonging to

Many neceſſary corrections of theſe Lemmata, as they appeared in Commandine's tranſlation, were made by Dr. Simson. Some of the corrections were ſupplied by the Pariſian MS. of Pappus, of which Dr. Moor procured a copy, as has formerly been mentioned; and they are remarked in the Doctor's reſtorations of the *Loci Plani*, and *Sectio Determinata*. In the Lemmata belonging to the Poriſms, which are all given in his poſthumous work on that ſubject, and in an improved ſtate, are many important emendations; but it does not appear

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that work were aſſumed by him, in the manner now mentioned; and Dr. Halley, in his edition of Apollonius, gives theſe Lemmata from the Savilian MSS. of Pappus. In the demonſtrations of the Propoſitions of Apollonius, Dr. Halley refers occaſionally to thoſe Lemmata, as the want of them appeared in the text for rendering the demonſtrations compleat. It appears likewiſe from the ſame edition, (p. 97 of the tranſlation of vth, vith, and viith, books) that Abdolmelec Schirazita, an Arabian epitomizer of the *Conics*, had collected ſome other Lemmata, (8.) beſides thoſe in Pappus, which were aſſumed without proof in the demonſtrations of the ſeventh book, and which therefore he prefixes to that ſeventh book; and they are placed by Dr. Halley immediately after the Lemmata of Pappus belonging to it.

In modern times, however, a Lemma is underſtood to be an eaſy preliminary Propoſition, which might have been incorporated with the more important one to which it is prefixed, but is more conveniently detached from it, and ſometimes is itſelf a Propoſition deſerving notice. In the treatiſe of Archimedes on the Sphere and Cylinder (before Prop. 17 b i.) is an eaſy Lemma demonſtrated by himſelf; and at the end of Prop. 17th are ſeveral Lemmata, which he ſtates to have been demonſtrated by thoſe who preceded him; and are Propoſitions of the xiith b. of Euclid. They are in ſubſequent Propoſitions aſſumed as known, but a formal quotation of preceding works was not the practice of the ancient geometers.

that any of theſe laſt were derived from that MS. as he never quotes it ; and moſt of them, indeed, were made before Dr. Moor obtained that copy of the ſeventh book.[‡] A conſiderable number of the Lemmata of the ſeventh book (about 90) have been publiſhed, as corrected by Dr. Simson ; and the attention and accuracy with which he has examined theſe Propoſitions, will be obvious to the intelligent reader, by comparing them with the edition of them in Commandine's tranſlation. And though the remaining Lemmata in this book do not appear to have been ſo particularly conſidered by the Doctor, yet there are in his notes on them many neceſſary corrections, with uſeful explanations, ſome of which have been mentioned in the notes added to this memoir.

The Lemmata in this ſeventh book belong to the ſeveral treatiſes deſcribed in the preface ; according to the annexed arrangement from Commandine's tranſlation ; in which enumeration, however, are ſome irregularities, as is obſerved by Dr. Simson.

[‡] In Dr. Simson's Pappus is a memorandum for enquiry if the copy of the *Codex Regius* contained any of the neceſſary emendations which he had made on a Lemma (Prop. 130.) of the Poriſms, but no mention is made of the reſult; which is a further preſumption that the Doctor, though he had had the uſe of the MS. for correcting the Lemmata of the two treatiſes above mentioned, yet that he had not had an opportunity of conſulting it reſpecting the Poriſms.

Treatiſes.				No. of Lemmata.
No. 1.	Apollonius,	*De Sectione Rationis et Spatii,* from beginning to Prop. 21 incl.		21
No. 2.	———	*De Sectione Determinata,*	to Prop. 64.	43
No. 3.	———	*De Inclinationibus,*	to — 95.	31
No. 4.	———	*De Tactionibus,*	to — 118.	23
No. 5.	———	*De Locis Planis,*	to — 126.	8
No. 6.	Euclides,	*De Porismatis,*	to — 164.	38
No. 7.	Apollonius,	*De Conicis,*	to — 234.	70
No. 8.	Euclides,	*De Locis ad Superficiem,*	to — 238.	4*
				238

Of theſe Lemmata, ſince Commandine's tranſlation, have been publiſhed:

By Dr. Halley, Greek and Latin,	No. 1, and 7. - -	91
By Dr. Simson, in Latin, and corrected,	Nos. 2, 5, and 6.	89
By Camerer, Greek and Latin,	No. 4. - - - -	23
		203

A few others have been mentioned in this Memoir as corrected by Dr. Simson and others.

It is proper alſo to mention in this place, that a Propoſition of ſome curioſity, though not in Commandine's tranſlation, is found at the end of this book in the two Pariſian MSS. and alſo in thoſe belonging to the Savilian Library at Oxford. It is entitled λῆμμα τȣ̃ ἀναλυομένȣ τόπȣ; and an accurate copy of it, from the Pariſian MSS. is placed in Dr. Simson's Pappus, from which a tranſlation by the Doctor is added.

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* Four only in Commandine's tranſlation are numbered as Propoſitions, though it ſeems that five Lemmata were intended. And it is alſo to be remarked, that the four laſt lines of fol. 300. a. Command. though printed in the character of the Commentary, are part of the text, of which the Greek is to be found in Savil. MS. No. 3. Dr. Simson, however, does not appear to have conſidered theſe Lemmata; at leaſt he has left no notes reſpecting them. See the Memoir, p. 48.

" Propoſitio Pappi Alexandrini quam ex Cod. MSS.† " Bibliothecæ Pariſienſis paucis abhinc Septimanis exſcripſit " D^{nus} Jacobus Moor, collega meus, 7mo Nov^{ris} 1748.

" Si latera circa angulum rectum trianguli BAC, (Fig. 10.) " viz. BA AC, ſecentur in K, L, ita ut tam BK ad KA quam AL " ad LC ſit ut BA ad AC, et jungantur BL CK, ſibi mutuo " occurrentes in F; et ad baſim BC ducatur AFG, erit AG ad " BC perpendicularis.

" Ducatur enim CE parallela ipſi BA occurratque ipſis BF, " AG, (productis) in E, H; et puta verum eſſe theorema, ſc. " angulum AGB rectum eſſe: æquiangula igitur ſunt BAC " ACH triangula, quia ABC angulus æqualis eſt ipſi GAC ſeu " HAC (8 6.) et recti ſunt anguli BAC, ACH; ut igitur BA ad " AC, ita AC ad CH. Eſt autem propter parallelas EC ad CH " ut (BK ad KA hoc eſt ex hypotheſi, ut BA ad AC hoc eſt ut) " AC ad CH: æquales igitur ſunt EC, CA. Et propter parallelas " eſt AL ad LC ut BA ad CE, hoc eſt ad AC, quod quidem " verum eſt ex hypotheſi.

" Componetur vero ita. Eadem manente conſtructione, " quoniam ex hypotheſi eſt BA ad AC ut AL ad LC, hoc eſt " ut BA ad CE; erunt AC, CE, æquales. Eſt itidem ex hypo- " theſi BA ad AC ut (BK ad KA hoc eſt ut) CE ſeu AC ad " CH: ergo (6. 6.) æquiangula ſunt triangula BAC, ACH, et " angulus ABC æqualis ipſi HAC ſeu GAC, et in triangulis ABC, " GAC communis eſt angulus ACB, ergo reliquus AGC æqualis " eſt reliquo BAC, ſc. angulo recto. Q. E. D.

" N. B. In Prop. Pappi facile oſtenditur AK, AL, equales eſſe."

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† " Viz. No. 2368, et No. 2440.

In the ſame MS. volume from which the preceding Propoſition is copied, there is the following different ſtatement of it:

[The ſame figure.]

“ Si ſit triangulum ABC rectum habens angulum BAC, “ ducatur autem BD parallela ipſi AC æqualis autem rectæ “ BA; et CE parallela ipſi BA æqualis autem ipſi AC; et CD “ BE jungantur ſibi mutuo occurrentes in F, et juncta AF “ occurrat baſi BC in G, erit AG perpendicularis ad BC.”

This Propoſition, which is equivalent to the preceding, was ſuggeſted by the figure of the 47 Prop. 1. Eucl. and had been demonſtrated by Dr. Simson, before he met with the other. There is no intimation in the manuſcripts hitherto examined of the purpoſe for which this Propoſition was placed at the end of this book; and though it is called a lemma of the *τόϖος ἀναλυομένος*, there is no apparent connection between it and any of the treatiſes deſcribed in this book, or with any of the *Lemmata* belonging to them; but from its being found in ſo many manuſcripts of Pappus, there is a preſumption of its having been placed in the *Mathematical Collections* by him.

§. VII. *The Eighth Book.*

The object of this laſt book* is to give ſome account of the ancient mechanics; and though a curious document of the ſtate of that branch of ſcience in the time of Pappus, yet from the great improvement both in the theory and practice of mechanics in modern times, it is comparatively of little value.

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* The title from MS. Bull. is “ Πάϖϖȣ ’Αλεξανδρέως συναγωγῆς ῆον ϖεριέχει δὲ “ μηχανικὰ ϖροβλήματα σύμμικτα§ ἀνθηρά.”

§ σύμμικτα is wanting in MS. Savil. No. 3.

The original genius of ARCHIMEDES was diſtinguiſhed in this department, as appears in part even from the portions of his numerous writings which have been preſerved, and alſo from the many references to his mechanical inventions in this book of PAPPUS, and in many other ancient writers. A conſiderable part of this book is employed in deſcribing what were then, and ſtill are, called the Mechanic Powers, and the moſt obvious combinations of them for the common purpoſes of life; eſpecially for raiſing up and for drawing very great weights.* PAPPUS avows its being chiefly borrowed from HERO, a diſtinguiſhed geometer and mechanician, whoſe works he frequently quotes, and ſome of which ſtill remain.† There is a long preface, which from ſome ſtatements of the mechanical notions, and of the arts of that period, becomes curious. It contains alſo ſome obſervations on the utility of mechanics, and of the connection of mechanics with geometry; in it alſo are diſtinguiſhed ſeveral branches of that ſcience, with notices of treatiſes on it which are loſt. Beſides many references to HERO, there are very ample teſtimonies in this preface of the fame of ARCHIMEDES, from his mechanical writings and inventions,[‡] in many of which his geometrical and arithmetical ſcience was employed. But as has already been remarked, the chief value of this book is from the information which it affords of the ſtate of mechanical ſcience in the age of Pappus. I muſt obſerve alſo, that though Dr. Simson makes a few corrections on this book, he ſeems never to have conſidered it particularly, and therefore it is unneceſſary to give a more particular account of its contents.[§]

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* Towards the end of the book are deſcriptions of the ancient machines for theſe purpoſes; and in the MS. copies are deſigns of ſuch machines made from theſe deſcriptions. It may be obſerved, however, that thoſe drawings are different from each other, and from the engravings in COMMANDINE, which no doubt had been copied from ſketches in the MS. which he followed.

† This is the elder HERO, of Alexandria, who is ſuppoſed to have lived about fifty years after ARCHIMEDES.——GER. VOSSIUS.

§. VIII. *Concluſion.*

From the ſhort account which has been given of the ſeveral books of Pappus, the miſcellaneous nature of the Collection is ſufficiently apparent. Several ancient mathematicians are mentioned by him, of whom no other notice remains; and many curious particulars are detailed reſpecting the ancient ſtate of mathematics, and of the problems and theorems which engaged the attention of the geometers of thoſe times. It is written with only a general attention to method; there is little uniformity in the ſtile, and the work has probably been compoſed at different times. Some propoſitions are demonſtrated minutely, and rather diffuſely, while in others many important ſteps are omitted,* as is often remarked by COMMANDINE, and theſe omiſſions he has generally ſupplied. As a further proof of this character of the *Collections*, the very frequent repetitions in different parts of it, of the ſame Propoſitions, ſometimes with the ſame, and ſometimes with different demonſtrations, may be mentioned.§ It is, however, a moſt intereſting work, and more eſpecially from the account of the analytical geometry of the ancients in the ſeventh book; an edition of the Greek, from a collation of the many manuſcripts which are known ſtill to exiſt in the libraries of Europe, would be moſt acceptable to all the admirers of the elegance and accuracy of ancient geometry; and for ſuch an undertaking Dr. SIMSON's notes would be highly valuable.‡ He does not ſeem to have taken a regular and accurate ſurvey of the whole, with any view to the immediate completion of ſuch a work; nor has he conſidered every paſſage which would require correction or explanation; yet his emendations and illuſtrations, ſeveral of which have been mentioned in this Memoir, will be an important repoſitory of materials for the aſſiſtance of any future editor.

[‡] In Prop. 10th, he mentions the well-known obſervation of Archimedes, that with a fixed ſtation he could move the earth. And the reſolution of the general mechanical problem, "to move a given weight with a given power," he calls the 40th invention of Archimedes.

[§] It is proper, however, to mention an obſervation of Dr. Simson on Prop. 14 of this book: "Addatur hic (poſt verbum *quidem* in linea quarta hujus paginæ, viz. fol. 320. b.) "*Geometrice* ut dudum emendavit Gregorius a St°. Vicentio, pag. 291." Q*uad. Circuli.*——In the ſame place is alſo the following note, referring to the *Codex* Bull.; he ſays, "In quo poſt verba ὀργανικῶς εὑρεῖν, legitur μεθοδεύεται δὲ τὸν τρόπον τοῦτον. "Verbo vero μεθοδεύεται ſignificatur conſtructio geometrica." Gregory alſo makes another remark on Commandine's Note B. in the ſame page of Pappus, (1588.) The laſt ſentence of the enunciation of Prop. 12, (fol. 317. a.) in Commandine's tranſlation is "Invenitur autem methodo inveſtigata hoc pacto;" but it ought to have been "inveſtigabitur autem geometrice ita;" the Greek (MS. Bull.) being "εὑρίσκεται μεθοδευθὲν οὕτως." Dr. Simson alſo remarks, reſpecting the laſt ſentence of Commandine's Note B. on Prop. 14, viz. 'Mirum eſt Pappum,' &c. "Non mirum "quoniam ex elementis conicis ſatis hujus demonſtratio patet."

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* Dr. SIMSON makes a remark to this purpoſe, *Opera Reliqua*, p. 526, at the end of Prop. 76. Many of the imperfections of PAPPUS, as they appear in the exiſting MSS., muſt be attributed to the tranſcribers.

§ The following examples may be remarked. The two ſtatements of the ancient claſſification of problems, in b. iii. fol. 4. b. and b. iv. fol. 60. The two accounts of the Conchoid in theſe two books. The two ſolutions of the Deliacal Problem, b. iii. fol. 7, and b. viii. Prop. 2, nearly in the ſame words. In Dr. SIMSON's edition of the *Sectio Determinata*, ſeveral duplicates are mentioned. Other repetitions alſo might be pointed out, ſuch as Props. 31, 57, 178, 194, book vii. Alſo Props. 87, 153, 206, in the ſame book. Alſo Props. 23, 58, 193, b. vii. Several of theſe are remarked by Dr. SIMSON, in his notes, both printed and unpubliſhed. See *Opera Reliqua*, pp. 409, 410.

It is proper likewiſe to obſerve, that among Dr. SIMSON's notes are ſeveral generalizations of Propoſitions in PAPPUS, and alſo ſome connected Propoſitions, which, though valuable in themſelves, may probably not be all conſidered as properly belonging to a new edition of that author. Several alſo of the Doctor's notes contain corrections of COMMANDINE's commentary, moſt of which might be omitted in conducting a new edition; of which the proper object muſt be, to give the text of PAPPUS free from errors, and to introduce ſuch comments only as are neceſſary for explaining real difficulties, and for filling up material deficiencies in the demonſtrations.

It may be alſo obſerved, that from what is known reſpecting ſeveral of the remaining MSS. and the ſimilarity of their defects, it is much to be apprehended that few things of material importance, beyond what Dr. SIMSON has already remarked, are likely to be diſcovered; though an accurate examination of theſe MSS. would certainly be moſt deſirable.

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‡ By comparing Dr. SIMSON's MS. notes on the Lemmata of the ſeventh book with his corrected edition of theſe Lemmata in his reſtorations of the *Loci Plani Sectio Determinata*, and *Porisms*, his taſte and judgment may be remarked in ſelecting only thoſe notes which were neceſſary, and omitting the others which he had written on his firſt review of theſe Propoſitions.

After every aid is obtained from remaining MSS. the ability and intelligence of an editor muſt be depended on for the beſt uſe of exiſting materials; and it is to be conſidered, that, in geometry, the nature of the ſubject, and the train of argument, may often enable an editor to ſupply the defects and correct the errors of manuſcripts, to which, indeed, thoſe on mathematical ſubjects appear to be more liable than any others.

The copy of COMMANDINE's PAPPUS which Dr. SIMSON uſed, and in which he wrote his notes, was the firſt edition PISAURI, 1588, and VENETIIS, 1589, and to this all quotations in this memoir refer.

It is neceſſary alſo to mention, that a few years after the Doctor's death, an application was made to his executor Mr. CLOW, on the part of the Delegates of the Clarendon Preſs, in the Univerſity of Oxford, for a tranſcript of all the MS. notes in this volume, as they were left by the Doctor. This was readily complied with, on the reaſonable condition, that whatever notes and corrections of Dr SIMSON's might be adopted in a new edition of PAPPUS, they ſhould be particularly diſtinguiſhed and acknowledged as his. Some time after, Mr. CLOW depoſited this valuable *legacy* from his deceaſed friend, in the library of the College of Glaſgow ; and having had, by the favour of the Principal and Profeſſors, every convenience for conſulting it, I have been enabled to give from it ſome intereſting particulars of Dr. SIMSON's geometrical ſtudies.

§. IX. *Of the MSS. of* PAPPUS.

A Conſiderable number of manuſcripts of the collections of PAPPUS are to be found in various libraries of Europe, but all of them which have been examined, are mutilated; and contain many errors, from the ignorance or careleſsneſs of the tranſcribers.[†]

COMMANDINE appears to have had the uſe of only one manuſcript, of which the hiſtory and fate are unknown; but from his numerous corrections of it in his commentary, in which he generally inſerts the erroneous paſſage of the MS. a judgement may be formed of its very deficient ſtate. In it, as in ſeveral other MSS.[‡] the two firſt books are entirely wanting.

In ſeveral MSS. however, a portion of the ſecond book (about one half) is preſerved. This was found in one of the Savilian MSS. viz. No. 9, and publiſhed by Dr. WALLIS, in 1688, with learned notes. It is underſtood alſo, that one of the Pariſian MSS. No. 2368, has this portion of the ſecond book, beginning at the ſame words as the fragments in the Savilian MS.[‡] From a MS. of **Pappus**, which belonged to **Joseph Scaliger**, the portions of the preface to the viith book, deſcribing the *Sectio Rationis* and *Sectio Spatii*, were publiſhed in Greek by **Snellius**, in his reſtoration of theſe books, in 1607; and his edition of the Greek preface to the *Sectio Determinata*, 1608, was probably from the ſame MS.

[†] Dr. SIMSON, in a note on a Propoſition, of the ſeventh book of PAPPUS, remarks, " non pauca autem in hac Propoſitione, in eo codice (Pariſienſe Regio ſc.) " vitiata ſunt, ut in omnibus fere PAPPI Propoſitionibus, et, ut videtur, in omnibus " manuſcriptis."

[‡] The ſame is to be remarked of the Savil. MS. No. 3; alſo of a MS. of PAPPUS in the Neapolitian Royal Library, mentioned by FABRICIUS *Bibl. Græc.* tom. v. page 790, Hamburgh, 1798, &c. In the MS. BULLIALDI, afterwards to be mentioned, the two firſt books alſo are wanting. GER. VOSSIUS, *de Scientiis Math.* cap. xvi. 7, when ſtating the books of the *Mathematical Collectionss* adds, " ſed duo primi videntur " deperiſſe."

The MS. of **Pappus** which was in the library of **Isaac Vossius**, and was carried from England to the Univerſity of Leyden, had alſo about one half of the ſecond book[*]; and probably, the preciſe portion which is in the other MSS. now mentioned.

In the edition of the *Bibliotheca Græca*, publiſhed by **Harles**, vol. ix. p. 170, is a catalogue of the MSS. of **Pappus** known to that writer, including thoſe now mentioned and ſeveral others. I ſhall in this place therefore mention only two more, not taken notice of by him. In the year 1795, J. **Gugl. Camerer** publiſhed an edition of the **Apollonius Gallus** by **Vieta**; and prefixes to it the preface and Lemmata belonging to the Tactions, in Greek. For this purpoſe he made uſe of three MSS. the two in the Pariſian library No. 1440, and No. 2368; and another in the Straſburgh library, not mentioned in Fabricius; but from the very ſhort references to that MS. nothing can be pronounced reſpecting its hiſtory and value.

[‡] A MS. copy of this fragment of the second book, (with the third book alſo,) taken from this Pariſian MS. is in the Savilian Library at Oxford; and from a note at the end of this copy, it appears that the Pariſian MS. was written in 1562.

[*] See the account of **Isaac Vossius** in the *Biog. Brit.* This MS. is mentioned in the Oxford Catalogue of Manuſcripts, No. 2126.

There is alſo at Edinburgh an elegant manuſcript of five books of Pappus, viz. the third, fourth, fifth, ſixth, and eighth; but unfortunately the ſeventh, the moſt valuable portion of the work, is wanting.

This MS. was purchaſed at Paris, in 1748, by Dr. James Moor, then Greek profeſſor at Glaſgow, from Mr. De Mairan, of the Academy of Sciences; and on the firſt page of it is written *D'Ortous de Mairan*, probably by his own hand. De Mairan had informed Dr. Moor that it had belonged to Bullialdus; and on that account Dr. Simson, who had it for ſome time in his poſſeſſion, and took many notes from it, calls it *Codex* Bullialdi.[†] Like the other MSS. of Pappus it abounds with errors; but Doctor Simson obtained from it ſeveral improved readings and corrections, which he has remarked in his copy of Commandine's tranſlation. Some of them are curious and important, and all merit the attention of any future editor of that work.

[†] Dr. Simson, in his copy of Commandine's Pappus, has the following notices of this MS. which, without doubt, he had from Dr. Moor; and which may therefore be conſidered as authentic. In fol. 320. b. (Pappi, edit. Pis. 1588.) "3^{tio} Nov^{ris} A. D. " 1748. Predictam Gregorii (a Sanct. Vinc.) emendationem veram eſſe, oſtendit " codex manuſcriptus elegantiſſimus, quem tribus abhinc diebus Pariſiis huc attulit " D. Jacobus Moor, collega meus doctiſſimus, &c."

Alſo in fol. 1. b. Pappi, " Hiſce litteris adſcriptis emendavimus tum ſchema " Commandini tum MS^{ti} Græci quem Dom. *D'Ortous de Mairan* dicit fuiſſe " Bullialdi. R. S."

In a detached paper, referring to fol. 77. b. Pappi, is a note in Dr. Simson's hand, " 25 Martii 1750, inveni hæc ita eſſe in codice Græco qui Bullialdi fuerat, " quemque collega meus D. Jacobus Moor, a domino Mairan emptum Pariſiis huc " (Glaſguam) attulit."

Some peculiarities in this MS. may be mentioned, for the ſatisfaction of thoſe who may be inclined to conſult it.

At the end of the third book are five pages of MS. not in COMMANDINE's edition, which contain ſome variations of the ſolution of the Deliacal problem, in addition to thoſe contained in the beginning of that book.§

At the end of the fourth book is a very imperfect ſketch of a Propoſition, not in COMMANDINE, which has already been mentioned in the preceding account of this fourth book.

In this MS. are many corrections on the margin, and ſome even in the text, by a later hand; and among them are a great number of the emendations propoſed by COMMANDINE of the very ſame errors, in the MS. uſed by him, from which a connection between theſe two MSS. may be inferred. Many of theſe emendations are ſervilely copied, retaining even the miſtakes into which COMMANDINE, in a few of them, had fallen.

This MS. though elegantly written, has been copied by a perſon totally ignorant of the ſubject; of which the number of groſs errors in its firſt ſtate is a ſufficient proof. It may be remarked, alſo, that at the beginning of a Propoſition, or of a paragraph, there is uſually a red letter, and a new line; but frequently alſo this diſtinction is made in the middle of a ſentence, while the beginning of Propoſitions and ſubjects in other places is not diſtinguiſhed in that or in any other manner. The enumeration of the Propoſitions is often irregular, and many of the diagrams, though neatly drawn, are altogether erroneous. The MS. was ſold at Dr. Moor's death, and was afterwards purchaſed for the library of the Faculty of Advocates at Edinburgh, in which the writer of this Memoir had every facility for conſulting that MS. and other books in that great collection. From the well-known liberality of the Curators of that eſtabliſhment, the aid of this MS. will, without doubt, be moſt readily given, when it can be uſeful for preparing an edition of that intereſting work in the original language.

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§ To theſe additional pages the following title is prefixed, "ἄλλως.—το δέκατον "θεωρήμα ἐν τῶ τρίτῶ τῆς τοῦ Πάππου συναγωγῆς καὶ την ἀποδέιξιν περιεχον, καὶ την ὀργάνικην "κατασκέυην τοῦ τε διπλασιασμοῦ τοῦ κύβου και τῶν δύο μεσῶν ἀνάλογον."—It may be obſerved that the SAVIL. MS. No. 3, has a ſimilar addition at the end of the 3d book.

This MS. not having the ſeventh book, Dr. Moor procured a copy of it to be taken from the MS. No. 2368, in the Pariſian library. This alſo for ſome time was in Dr. Simson's poſſeſſion, and he adopts from it ſome corrections of Commandine's tranſlation in his reſtoration of the *Loci plani*, and *Sectio determinata*; but, as was already mentioned, there is no reference to it in his expoſition of the Poriſms[‡]. I add with regret, that this copy, in the diſperſion of Dr. Moor's library, ſeems to have been loſt; and theſe particulars are ſtated, as they may perhaps facilitate the recovery of it from ſome obſcure ſituation into which it may have accidentally fallen.

[‡] In a note on Commandine's Pappus, the Doctor gives the following account of this tranſcript, which he muſt have got from Dr. Moor "Hunc autem librum 7^{mum} Pappi ex eo codice (ſcil. No. 2368, Reg. Bibl. Par.) deſcripſit Dom Caperonier, linguæ Græcæ in Academia Pariſienſi Profeſſor; ſumptibus D. Jac. Moor, collegæ mei doctiſſimi. Schemata vero ejus depinxit D. Joa. Brisbane, M. D."

The two Savilian MSS. No. 3, and No. 9, have been repeatedly referred to. There is alſo in that library a tranſcript from the Pariſian MS. No. 2368, of a portion of the ſecond book of Pappus, (the ſame as the fragment in MS. Savil. No. 9, publiſhed by Dr. Wallis,) and the whole of the third book. This tranſcript is not mentioned by Dr. Wallis; and in the following Appendix II. it is quoted as MS. Savil. B. From a note at the end, it appears that the Pariſian MS. No. 2368, was written in 1562, by the direction of P. Ramus.

APPENDIX II.

TWO paſſages of PAPPUS, on the ancient diviſion of geometrical lines into claſſes, have, in the preceding Memoir, been repeatedly alluded to. For the ſatisfaction of those who may wiſh to conſider them accurately, the Greek, from the SAVILIAN MSS. and the MS. BULL. is printed in this Appendix, along with COMMANDINE's tranſlation. Some of the various readings in theſe MSS. are remarked, but without taking notice of either the ſmaller differences, which do not affect the ſenſe, or of ſome groſs errors, manifeſtly ariſing from the careleſſneſs of the tranſcribers.

Some of COMMANDINE's notes are alſo mentioned; but as Dr. SIMSON has not left any obſervations on theſe paſſages, it is not my purpoſe to enter into any examination or explanation of them, except only by pointing out the references to ſome Propoſitions, not explicitly ſtated either in PAPPUS, or in the commentary of his tranſlator.

PAPPI, MS. Savil. *No.* 3. *Fol.* 10, *b.*

“Ἃ μὲν ἔδει με προειπεῖν ἐστι ταῦτα. Παρεὶς δὲ κρίνειν σοί τε καὶ ἄλλοις τοῖς ἐν γεωμετρίᾳ γεγυμνασμένοις τὰ ὑπ᾽ ἐκείνου προσγραφέντα‖ περὶ τῆς κατασκευῆς, καὶ τὰ ὑφ᾽ ἡμῶν ἐπενεχθέντα καλῶς ἔχειν ἡγοῦμαι, καὶ τὰ δόξαντα τοῖς ἀρχαίοις περὶ τοῦ προειρημένου προβλήματος ἐκθέσθαι. Καὶ πρῶτον εἰπεῖν ὀλίγα περὶ τῶν ἐν γεωμετρίᾳ προβλημάτων ἀρχὴν λαβὼν ἐντεῦθεν. Τῶν ἐν γεωμετρίᾳ προβλημάτων οἱ παλαιοί τρία γένη φασὶν εἶναι. Καὶ τὰ μὲν αὐτῶν ἐπίπεδα καλεῖσθαι, τὰ δὲ στερεὰ, τὰ δὲ γραμμικά. Τὰ μὲν οὖν δι᾽ εὐθείας καὶ κύκλου περιφερείας δυνάμενα λύεσθαι λέγοιτο ἂν εἰκότως ἐπίπεδα. Καὶ γὰρ αἱ γραμμαὶ δι᾽ ὧν λύεται τὰ τοιαῦτα προβλήματα, τὴν γένεσιν ἔχουσιν ἐν ἐπιπέδῳ. ὅσα δὲ προβλήματα λύεται παραλαμβανομένης εἰς τὴν γένεσιν μιᾶς τῶν τοῦ κώνου τομῶν, ἢ πλειόνων, ταῦτα στερεὰ κέκληται. πρὸς γὰρ τὴν κατασκευὴν ἀναγκαῖόν ἐστι χρήσασθαι στερεῶν σωμάτων† ἐπιφανείαις. Λέγω δὲ ταῖς κωνικαῖς. Τρίτον δ᾽ ἔτι καταλείπεται γένος, ὃ καλεῖται γραμμικόν.

“Quæ igitur me præmiſiſſe oportebat, hæc ſunt. Itaque omittens explicare et tibi, et iis, qui in geometria exercitati ſunt, ea, quæ ille ſcripſit de conſtructione, et quæ nos objecimus; optimum fore judicavi, ſi exponerem quid antiqui de dicto problemate ſenſerint: et primum nonnulla dicerem de problematibus, quæ in geometria conſiderantur, inde ſumpto initio.

“Problematum geometricorum antiqui tria genera eſſe ſtatuerunt, et eorum alia quidem *plana* appellari, alia *ſolida*, alia *linearia*. Quæ igitur per rectas lineas, et circuli circumferentiam ſolvi poſſunt, merito *plana* dicantur; etenim *lineæ*, per quas ejuſmodi problemata ſolvuntur, in plano ortum habent. Problemata vero quæcunque ſolvuntur, aſſumpta in conſtructionem aliqua coni ſectione, vel pluribus, *ſolida* appellantur; namque ad conſtructionem neceſſe eſt ſolidarum figurarum ſuperficiebus, nimirum conicis, uti. Reſtat tertium genus, quod *lineare* appellatur. Lineæ enim aliæ præter jam

‖ προγραφέντα, MS. B. Savil.

† σχημάτων, idem.

“γραμμαὶ γὰρ ἕτεραι παρὰ τὰς εἰρημένας εἰς τὴν κατασκευὴν λαμβάνονται ποικιλωτέραν καὶ μεταπλασμένην§ ἔχουσαι τὴν γένεσιν. ὁποῖαι τυγχάν* (ουσι αἱ ἕλικες) καὶ τετραγωνίζουσαι καὶ κογχλοειδεῖς,‡ καὶ κισσοειδεῖς, πολλὰ καὶ παράδοξα περὶ αὐτὰς ἔχουσαι συμπτώματα. Τοιαύτης δὴ τῆς διαφορᾶς τῶν προβλημάτων οὔσης, οἱ παλαιοὶ γεωμέτραι τὸ προειρημένον ἐπὶ τῶν δύο εὐθειῶν πρόβλημα τῇ φύσει στερεὸν ὑπάρχον, οὐχ οἷοίτε ἦσαν κατασκευάζειν τῷ γεωμετρικῷ λόγῳ κατακολουθοῦντες. ἐπεὶ μήδε τὰς τοῦ κώνου τομὰς ῥάδιο νὲν ἐπιπέδῳ γράφειν ἦν.” [ὡς δεῖ δύο δοθεισῶν εὐθειῶν ἀνίσων, δύο μέσας ἀνάλογον λαβεῖν ἐν συνεχεῖ ἀναλογίᾳ.] Τοῖς δὲ ὀργάνοις μεταλαβόντες αὐτὸ θαυμαστίως εἰς χειρουργίαν καὶ κατασκευὴν ἐπιτήδειον ἤγαγον. ὡς ἐστιν ἰδεῖν ἀπὸ τῶν φερομένων αὐτοῖς συνταγμάτων. λέγω δ' ἐν τῷ ἐρατοσθένους μεσολάβῳ, καὶ τοῖς φίλωνος καὶ ἥρωνος μηχανικοῖς, ἢ καταπαλτικοῖς. οὗτοι γὰρ ὁμολογοῦντες στερεὸν εἶναι τὸ πρόβλημα, τὴν κατασκευὴν αὐτοῦ μόνον ὀργανικῶς πεποίηνται, συμφώνως ἀπολλωνίῳ τῷ περγαίῳ. ὃς καὶ τὴν ἀνάλυσιν αὐτοῦ πεποίηται διὰ τῶν τοῦ κώνου τομῶν. Καὶ ἄλλοι διὰ τῶν ἀρισταίου τόπων στερεῶν,

“dictas in conſtructionem aſſumuntur, varium, et tranſmutabilem ortum habentes, quales ſunt helices, et [quas Græci τετραγωνίζουσας appellant, nos] quadrantes [dicere poſſumus,] conchoides, et ciſſoides, quibus quidem multa, et admirabilia accidunt. Cum igitur tales ſint problematum differentiæ, antiqui geometræ problema ante dictum in duabus rectis lineis, quod natura ſolidum eſt, geometrica ratione innixi conſtruere non potuerunt; quoniam neque coni ſectiones facile eſt in plano deſignare: inſtrumentis autem ipſum in operationem manualem, et commodam, aptamque conſtructionem mirabiliter traduxerunt, quod videre licet in eorum voluminibus, quæ circumferuntur, ut in ERATOSTHENIS meſolabo, in PHILONIS, et HERONIS mechanicis et catapulticis. Hi enim aſſerentes problema ſolidum eſſe, ipſius conſtructionem inſtrumentis tantum perfecerunt, congruentur APOLLONIO Pergæo, qui et reſolutionem ejus fecit per coni ſectiones: alii per locos ſolidos ARISTÆI: nullus autem per ea, quæ proprie

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§ βεβιασμένην, idem.

* In MS. Savil. No. 3, quæ inter uncos, rubro liquore exaratæ a secunda manu.—In altero codice Savil. “αἱ ἕλικες” absunt. In MS. BULL. τυγχάνουσι αἱ . . . . καὶ.

‡ κογχοειδεῖς, MS. B. SAVIL.

‖ Quæ inter uncos, in COMMANDINI verſione omittuntur; ſed quamvis ſupervacanea, forſan non erronea.

“ ὀυδεὶς δὲ διὰ τῶν ἰδίως ἐπιπέδων καλουμένων.
“ Νικομήδης δὲ καὶ λόγῳ διὰ κογχλοειδοῦς*
“ γραμμικῆς, δι᾽ ἧς καὶ τὴν γωνίαν ἐτριχοτόμησεν.
“ ἐκθησόμεθα οὖν τέσσαρας αὐτοῦ κατασκευὰς,
“ μετά τινος ἐμῆς ἐπεξεργασίας. ὧν
“ πρώτην μὲν ἐρατοσθένειον, δευτέραν δὲ τὴν
“ τῶν περὶ νικομήδη, τρίτην δὲ τὴν τῶν περὶ
“ ἥρωνα μάλιστα πρὸς τὰς χειρουργίας ἁρμόζουσαν,
“ τοῖς ἀρχιτεκτονεῖν βουλομένοις. Καὶ
“ τελευταίαν τὴν ὑφ᾽ ἡμῶν ἀνευρημένην. στερεοῦ†
“ γὰρ παντὸς, ἕτερον στερεον ὅμοιον τῷ δοθέντι
“ κατασκευάζεται πρὸς τὸν δοθέντα λόγον.
“ ἐὰν δύο τῶν δοθεισῶν εὐθειῶν δύο καὶ‡ μέσαι
“ κατὰ τὸ συνεχὲς ἀνάλογον ληφθῶσιν, ὡς
“ ἥρων ἐν μηχανικοῖς καὶ καταπαλτικοῖς.

“ plana appellantur. At NICOMEDES,
“ et ratione illud fecit per lineam
“ conchoidem, per quam et angulum
“ tripartito diviſit. Exponemus igitur
“ quatuor ejus conſtructiones una cum
“ quadam noſtra tractatione. Quarum
“ prima quidem eſt ERATOSTHENIS,
“ ſecunda NICOMEDIS,
“ tertia HERONIS, maxime ad manuum
“ operationem accommodata,
“ iis qui architecti eſſe volunt. Ultima
“ autem eſt a nobis inventa: ſolido
“ enim quocunque dato, alterum ſolidum
“ dato ſimile conſtruitur ad
“ datam proportionem, ſi duabus
“ datis rectis lineis, duæ mediæ in
“ continua analogia aſſumantur, ut
“ inquit HERO in mechanicis et
“ catapulticis.

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* κογχοειδες, MS. SAVIL. B.

† στερεοῦ δεδομένου. MS. BULL. et ut videtur MS. quo utebatur COMMANDINUS.

‡ Deeſt καὶ MS. B. SAVIL. et MS. BULL.

PAPPI, MS. SAVIL. No. 3. Fol. 86, b.

“Τὴν δοθεῖσαν γωνίαν εὐθύγραμμον εἰς τρία ἴσα τεμεῖν οἱ παλαιοὶ γεωμέτραι θελήσαντες, ἠπόρησαν, δι' αἰτίαν τοιαύτην. τρία γένη φαμὲν εἶναι τῶν ἐν γεωμετρία προβλημάτων, καὶ τὰ μὲν αὐτῶν ἐπίπεδα καλεῖσθαι, τὰ δὲ στερεὰ, τὰ δὲ γραμμικά. τὰ μὲν οὖν δι' εὐθείας καὶ κύκλου περιφερείας δυνάμενα λύεσθαι, λέγοιτ' ἂν εἰκότως ἐπίπεδα. καὶ γὰρ αἱ γραμμαὶ δι' ὧν εὑρίσκεται τὰ τοιαῦτα προβλήματα τὴν γένεσιν ἔχουσιν ἐν ἐπιπέδω ὅσα δὲ λύεσθαι[*] προβλήματα, παραλαμβανομένης εἰς τήν γένεσιν μιᾶς τῶν τοῦ κώνου τομῶν, ἢ καὶ πλειόνων, στερεὰ ταῦτα κέκληται. πρὸς γὰρ τὴν κατασκευὴν χρήσασθαι στερεῶν σχημάτων ἐπιφανείαις, λέγω δὲ ταῖς κωνικαῖς, ἀναγκαῖον. τρίτον δέ τι προβλημάτων[†] ὑπολείπεται γένος, τὸ καλούμενον γραμμικὸν, γραμμαὶ γὰρ ἕτεραι παρὰ[‡] τὰς εἰρημένας εἰς τὴν κατασκευήν λαμβάνονται, ποικιλωτέραν ἔχουσαι τὴν γένεσιν, καὶ βεβιασμένην μᾶλλον, ἐξ ἀτακτοτέρων ἐπιφανειῶν, καὶ κινήσεων ἐπιπλεγμένων γενώμεναι, τοιαῦται δέ εἰσιν αἵ τε ἐν τοῖς πρὸς ἐπιφανείαις καλουμένοις τόποις εὑρισκόμεναι γραμμαὶ, ἕτεραί τε τούτων

“Antiqui geometræ datum angulum rectilineum tripartito ſecare volentes ob hanc cauſam hæſitarunt. Problematum, quæ in geometria conſiderantur, tria eſſe genera dicimus; et eorum alia quidem *plana*, alia *ſolida*, alia vero *linearia* appellari. Quæ igitur per rectas lineas, et circuli circumferentiam ſolvi poſſunt, merito dicuntur *plana*: *lineæ* enim per quas talia problemata inveniuntur, in plano ortum habent. Quæcunque vero ſolvuntur, aſſumpta in conſtructionem aliqua coni ſectione, vel etiam pluribus, *ſolida* appellata ſunt, quoniam ad conſtructionem ſolidarum figurarum ſuperficiebus, videlicet conicis, uti neceſſarium eſt. Relinquitur tertium genus problematum, quod *lineare* appellatur; lineæ enim aliæ, præter jam dictas in conſtructionem aſſumuntur, quæ varium et difficilem ortum habent, ex inordinatis ſuperficiebus, et motibus implicatis factæ. Ejuſmodi vero ſunt etiam lineæ, quæ in locis ad ſuperficiem dictis inveniuntur,

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[*] λυεται, MS. SAVIL. No. 9.

[†] *των προβληματων*, idem.

[‡] *προς*, idem.

“ ποικιλώτεραι, καὶ πολλαὶ τὸ πλῆθος ὑπὸ
“ δημητρίου τοῦ ἀλεξανδρέως ἐν ταῖς γραμμικαῖς
“ ἐπιστάσεσι, καὶ φίλωνος τοῦ τυανέως, ἐξ ἐπιπ-
“ λοκῆς πλεκ-οειδῶν§ τε καὶ ἑτέρων παντόιων
“ ἐπιφανειῶν εὑρισκόμεναι, πολλὰ καὶ θαυμαστὰ
“ συμπτάματα περὶ αὐτὰς ἔχουσαι. καὶ τινες
“ αὐτῶν ὑπό τῶν νεωτέρων ἠξιώθησαν λόγου
“ πλείονος. μία δέ τις ἐξ αὐτῶν ἐστιν, ἡ καὶ
“ ϖαράδοξος ὑπὸ τοῦ μενελάου κληθεῖσα γραμμή.
“ τοῦ δὲ αὐτοῦ γένους ἕτεραι ἕλικές εἰσι τετρα-
“ γωνίζουσαί τε, καὶ κοχλοειδεῖς καὶ κισσοειδεῖς.
“ δοκεῖ δέ πως ἁμάρτημα τὸ τοιοῦτον οὐ μικρὸν
“ εἶναι τοῖς γεωμέτραις, ὅταν ἐπίπεδον πρόβλημα
“ διὰ τῶν κωνικῶν ἢ τῶν γραμμικῶν ὑπό τινος
“ εὑρίσκεται. καὶ τὸ σύνολον ὅταν ἐξ ἀνοικείου

“ et aliæ quædam magis variæ, et
“ multæ a DEMETRIO ALEXAN-
“ DRINO [ἐν τᾶις γραμμικᾶις ἐϖιςάσεσι,
“ hoc eſt] in linearibus aggreſſioni-
“ bus ; et a PHILONE TYANEO ex
“ implicatione [ϖληκτοειδῶν,*] et aliarum
“ varii generis ſuperficierum inventæ,
“ quæ multa et admirabilia ſympto-
“ mata continent : et nonnullæ ipſa-
“ rum a junioribus dignæ exiſtimatæ
“ ſunt, de quibus longus ſermo habe-
“ retur. Una autem aliqua ex ipſis
“ eſt, quæ et admirabilis a MENELAO
“ appellatur.

“ Ex hoc genere ſunt lineæ helices,
“ et quadrantes, et conchoides, et ciſ-
“ ſoides : videtur autem quodam-
“ modo peccatum non parvum eſſe
“ apud geometras, cum problema
“ planum per conica, vel linearia ab
“ aliquo invenitur, et ut ſummatim

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§ ϖληκλοειδῶν. MS. BULL. et ut videtur MS. COMMAND.

* Πλεκλοειδων, or πληκλοειδων. This word, as appears from the application of it in Prop. 29. lib. iv. denotes a claſs of geometrical ſurfaces, produced by ſome preciſe rule, on which mathematical reaſoning reſpecting ſuch ſurfaces may be founded. By COMMANDINE, and alſo by VOSSIUS, they are ſuppoſed to have got this name from the complex motions by which they are produced. The *Plectoeides*, in Prop. 29, (ſee fig. of that Prop. in PAPPUS) is formed by the motion of the ſtraight line LKI paſſing through the ſtraight line BLN, to which it is always perpendicular, and alſo through the line produced by the common ſection of the conic ſuperficies, and of the ſuperficies called *Cylindroeides*, both mentioned in that Propoſition. See note G. p. 104 and 105.

Among the ancients, geometrical curves and figures generally got names from ſome reſemblance which they were ſuppoſed to have to common objects. Such are the *Conchoid* and *Ciſſoid*, and alſo the *Lunule*, or *Meniſcus*, the *Pelecoides*, the *Hippopeda*, and ſeveral others mentioned by PROCLUS. (See PROCL. on Def. 4. 8. 1 Elem.)——It is poſſible that ϖληκλοειδης may have had a ſimilar origin, not now to be traced ; but the etymology implied in COMMANDINE's tranſlation of it may be right.

“ γένους λύεται,|| οἷόν ἐστι τὸ ἐν τῷ ε̄ τῶν
“ ἀπολλωνίου κωνικῶν, ἐπὶ τῆς παραβολῆς πρό-
“ βλημα, καὶ ἐν τῷ περὶ τῆς ἕλικος ὑπὸ ἀρχι-
“ μήδους λαμβανομένη στερεὰ νεῦσις ἐπὶ κύκλου,
“ μηδενὶ γαρ προσχρώμενον στερεῷ, δυνατὸν εὑρεῖν
“ τὸ ὑπ᾽ αὐτοῦ γραφόμενον θεώρημα. λέγω δὴ
“ τὸ τὴν περιφέρειαν τοῦ ἐν τῇ πρώτῃ περιφορᾷ
“ κύκλου, ἴσην ἀποδεῖξαι τῇ πρὸς ὀρθὰς ἀγομένῃ
“ εὐθείᾳ τῇ ἐκ τῆς γενέσεως τῆς ἐφαπτομένης
“ τῆς ἕλικος. τοιαύτης δὴ τῆς διαφορᾶς τῶν
“ προβλημάτων ὑπαρχούσης, οἱ πρότεροι γεω-

“ dicam, cum ex improprio ſolvitur
“ genere, quale eſt in quinto libro
“ conicorum APOLLONII‡ problema
“ in parabola: et in libro de lineis
“ ſpiralibus ARCHIMEDIS: aſſumpta
“ ſolida inclinatio in circulo: fieri enim
“ poteſt, ut nullo utentes ſolido, pro-
“ blema ab ipſo deſcriptum invenia-
“ mus.† Dico autem circumferentiam
“ circuli in prima circulatione deſcrip-
“ tam demonſtrare æqualem rectæ
“ lineæ, quæ a principio lineæ ſpiralis
“ ad rectos angulos ducitur ei quæ eſt
“ circulationis principium, et a recta
“ linea ſpiralem contingente termi-
“ natur. Itaque cum hujuſmodi ſit
“ problematum differentia, antiqui

|| λυηται γενεσις, MS. SAVIL. No. 9.

‡ In COMMANDINE's time, the firſt four books only of APOLLONIUS's *Conics* had been diſcovered; and he thence makes no obſervation on this criticiſm of PAPPUS, reſpecting a Propoſition in the fifth book. ALEXANDER ANDERSON, however, in his *Exercitationum Mathematicarum Decas Prima*, (Paris 1619,) long alſo before the fifth, ſixth, and ſeventh books of APOLLONIUS were recovered from the Arabic, inferred from inveſtigations founded on the general deſcription of that fifth book by PAPPUS, that the problem about the Parabola, here alluded to by him, was truly a ſolid problem; and in *Exerc.* V. (page 24,) he ſtates it, and reſolves it. In the fifth book of APOLLONIUS we now find the firſt caſe of Prop. 58. is equivalent to the problem of ANDERSON.

† The Propoſition of ARCHIMEDES mentioned in this place is the 18th in his treatiſe *De Helicibus*. In that Propoſition the 7th is neceſſary, and in that 7th Propoſition (*De Helicibus*) the inclination alluded to is aſſumed.——COMMANDINE, in Note C. reſpecting this inclination, refers to Prop. 128. b. i. of VITELLO's *Optics*. That Propoſition is indeed a plane inclination, and is aſſumed in Prop. 5th of ARCHIMEDES's Treatiſe. But in Prop. 7. of the ſame treatiſe, which (as has been obſerved) is uſed in Prop. 18th, a ſolid inclination is aſſumed, the ſame as Prop. 130. b. i. of VITELLO, in which an hyperbola is required. See alſo ALHAZEN, b. v. Propp. 32, 33. In Prop. 42 of b. iv. of PAPPUS, is a *Locus ad Hyperbolam*, which he ſtates to have been aſſumed by ARCHIMEDES, in the ſolid inclination employed by him in his treatiſe *De Helicibus*.

" μετραι, τὸ προειρημένον ἐπὶ τῆς γωνίας πρό-
" βλημα, τῆ φύσει στερεὸν ὑπάρχον διὰ τῶν
" ἐπιπέδων ζητοῦντες, οὐχ οἷοι ἦσαν εὑρίσκειν,
" ὀυδέ πω‖ γὰρ* τοῦ κώνου τομὰι συνήθεις ἦσαν
" ἀυτοῖς. κὰι διὰ τοῦτο ἠπόρησαν. ὕστερον
" μέντοι διὰ τῶν κωνικῶν ἐτριχοτόμησαν τὴν
" γωνίαν εἰς τὴν ἕυρεσιν χρησάμενοι τῆ ὑπο-
" γεγραμμένῃ νεύσει."

" geometræ problema jam dictum in
" angulo, quod natura ſolidum eſt,
" per plana inquirentes invenire non
" potuerunt: nondum enim ipſis cog-
" nitæ erant coni ſectiones, et ob eam
" cauſam hæſitarunt. Poſtea vero
" angulum tripartito diviſerunt ex
" conicis, ad inventionem infra ſcripta
" inclinatione utentes.†

‖ ȣ δη πω, MS. SAVIL. No. 9.

* γας ἀι, MS. BULL. αι τȣ κωνȣ τομαι συνεχεις εισιν. MS. SAVIL. No. 9.

† Prop. 31, lib. iv. PAPPI.

APPENDIX III.

As the volume of Dr. Simson's poſthumous works is not in general circulation, it may be agreeable to the Mathematical reader, to ſee the Doctor's very improved tranſlation of the general deſcription of Euclid's Poriſms, in the preface to the ſeventh book of Pappus, annexed to the account of his labours. The mutilated detail of the contents of the three books of Poriſms, not being materially altered by Dr. Simson, except in a few Propoſitions reſtored by him, is not added.

"POST Tactiones in tribus libris habentur Poriſmata Euclidis, collectio artificioſiſſima multarum rerum quæ ſpectant ad analyſin difficiliorum et generalium problematum, quorum quidem ingentem copiam præbet natura. Nihil vero additum eſt iis quæ Euclides primum ſcripſerat, præterquam quod imperiti quidam qui nos præceſſerunt ſecundas deſcriptiones paucis ipſorum [ſc. Poriſmatum] appoſuerunt. Cum vero unumquodque definitum habeat demonſtrationum numerum,† ut oſtendimus,‡ Euclides unam eamque maxime evidentem in ſingulis poſuit. Habent autem ſubtilem et naturalem contemplationem, neceſſariamque et maxime univerſalem, atque iis quæ ſingula perſpicere et inveſtigare valent admodum jucundam Specie

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† "Forſan intelligit demonſtrationes diverſorum caſuum ejuſdem Poriſmatis; vel propoſitionum quæ ſunt ejus converſæ.

‡ "In quibuſdam ex lemmatibus, ni fallor, ad Poriſmata.

“ autem hæc omnia neque theoremata ſunt, neque problemata, ſed mediæ
“ quodammodo inter hæc naturæ, ita ut eorum Propoſitiones poſſunt vel ut
“ theoremata, vel ut problemata formari. Unde factum eſt, ut inter multos
“ Geometras alii hæc genere theoremata exiſtiment, alii vero problemata,
“ reſpicientes ad formam tantum propoſitionis. Differentias autem horum
“ trium melius intellexiſſe veteres manifeſtum eſt ex definitionibus. Dixerunt
“ enim theorema eſſe quo aliquid propoſitum eſt demonſtrandum ; problema
“ vero quo aliquid propoſitum eſt conſtruendum ; Poriſma vero eſſe quo aliquid
“ propoſitum eſt inveſtigandum. A Neotericis autem immutata eſt hæc
“ Poriſmatis definitio, qui hæc omnia inveſtigare haud potuerunt, ſed
“ Elementis hiſce adhibitis, oſtenderunt tantum quid ſit quod quæritur, non
“ autem illud inveſtigaverunt. Et quamvis a definitione et ab ipſis rebus quæ
“ traditæ ſunt redarguerentur, hoc tamen modo, ab accidente, definierunt.
“ Poriſma eſt quod deficit hypotheſi a Theoremate Locali [hoc eſt, Poriſma eſt
“ Theorema Locale deficiens ſive diminuta in hypotheſi ejus.] Hujus autem
“ generis Poriſmatum Loca Geometrica ſunt ſpecies, quorum magna eſt copia
“ in Libris de Analyſi ; ac ſeorſim a Poriſmatibus collecta, ſub propriis titulis
“ traduntur, eo quod magis diffuſa et copioſa ſit hæc præ cæteris ſpeciebus. E
“ Locis enim quædam plana ſunt quædam ſolida, quædam linearia, et præter
“ hæc ſunt Loca ad medietates [ſive a mediis proportionalibus orta.] Accidit
“ hoc etiam Poriſmatibus, Propoſitiones habere conciſas propter difficultatem
“ multarum rerum quæ ſubintelligi ſolent; unde evenit Geometras non paucos
“ ex parte tantum rem perſpicere, dum ea quæ inter oſtenſa magis neceſſaria
“ ſunt haud capiunt. Multa autem ex iis in una Propoſitione minime com-
“ prehendi poſſunt, quia ipſe Euclides non multa in unaquaque ſpecie
“ poſuerit, ſed ut ſpecimen daret multæ copiæ, pauca ad principium primi
“ libri poſuit ejuſdem omnino ſpeciei cum uberrima illa [quæ in primo libro
“ habetur] Locorum ſpecie, ut decem ſint numero. Quare has [Propoſitiones
“ ſcilicet hujus ſpeciei] una Propoſitione comprehendi poſſe animadvertentes,
“ eam ita deſcribimus.

‘ Si quadrilateri cujus anguli oppoſiti vel ex adverſo, vel ad eaſdem partes
‘ ſunt poſiti,* [lateribus productis] data ſint in uno ipſorum tria puncta

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* “ Primum horum Pappus vocat ὕπτιον, alterum παρύπτιον, de quibus vide Not. C.
“ pp. 85, 86, 87.

‘ [interſectionum ſcilicet ;] vel ſi in quadrilatero cujus duo latera ſunt inter ſe ‘ parallela data ſint duo puncta interſectionum in altera parallelarum ;] cætera ‘ vero puncta præter unum tangant rectam poſitione datam ; etiam hoc tanget ‘ rectam Poſitione datam.’ “ Hoc autem de quatuor tantum rectis dicitur “ quarum non plures quam duæ per idem punctum tranſeunt. In quolibet “ vero propoſito rectarum numero ignoratur, quamvis vera ſit hujuſmodi “ Propoſitio, viz.

‘ Si quotcunque rectæ occurrant inter ſe, nec plures quam duæ per idem ‘ punctum ; data vero ſint puncta omnia in earum una, unumquodque autem ‘ punctum in alia tangat rectam, poſitione‖ datam.’ Vel generalius ſic. ‘ Si ‘ quotcunque rectæ occurrant inter ſe, neque ſint plures quam duæ per idem ‘ punctum, omnia vero puncta [interſectionem ſcilicet] in earum unâ data ſint ; ‘ reliquorum numerus erit numerus triangularis, cujus latus exhibet numerum ‘ punctorum rectam poſitione datam tangentium ; quarum interſectionum ſi ‘ nullæ tres exiſtant ad angulos trianguli ſpatii nullæ quatuor ad angulos ‘ quadrilateri, nullæ quinque ad angulos quinquelateri, &c. i. e. Univerſim, ſi ‘ nullæ harum interſectionum in orbem redeant] unaquæque interſectio reliqua ‘ tanget rectam poſitione datam.’

“ EUCLIDEM autem hoc neſciviſſe haud veriſimile eſt, ſed principia ſola “ reſpexiſſe : nam per omnia Poriſmata non niſi prima principia, et ſemina “ tantum multarum et magnarum rerum ſparſiſſe videtur. Hæc autem juxta “ hypotheſium differentias minime diſtinguenda ſunt ; ſed ſecundum differentias “ accidentium et quæſitorum. Hypotheſes quidem omnes inter ſe “ differunt, cum ſpecialiſſimæ ſint : accidentium vero et quæſitorum unumquodque, “ cum ſit unum idemque multis diverſisque hypotheſibus contingit.§

“ Talia itaque inquirenda offeruntur in primi libri Propoſitionibus ; (in “ principio ſeptimi habetur* diagramma huc ſpectans) ‘ Si a duobus punctis ‘ datis inflectantur duæ rectæ ad rectam poſitione datam, abſcindat autem ‘ earum una à rectâ poſitione data ſegmentum dato in ea puncto adjacens, ‘ auferet etiam altera ab aliâ rectâ ſegmentum datum habens rationem.’ i. e. “ *Quod ad alterum ſegmentum habebit rationem eandem rationi quæ ex* “ *hypotheſi data eſt.*” Deinde in ſubſequentibus ; &c.”

‖ “ Unum tangat unam, aliud tangat aliam rectam poſitione datam, et ſic deinceps.”

§ “ Ex gr. Multa ſunt Poriſmata quæ diverſas hypotheſes habent, ſed quæ omnia “ concludunt punctum aliquod tangere rectam poſitione datam ; vel rectam aliquam vergere “ ad punctum datum, &c.”

* “ Non jam habetur.”

ERRATA.

Page 18, line laſt of note, for 87 read 37
38, l. 6, note, for *rationum* read *rationem*
40, l. 5, note, for 280 read 270
89, l. 5, after μένα place a period.
90, l. laſt of note, for γ read γ̄ (3)
91, l. dele note, "*forſan* ἡδομενοις"
94, l. 1 note, for *indeterminatum* r. *indeterminatus*
114, l. 22, for *letter after*, read *letter, after*
121, l. 4, for *into ; it* read *into it ;*

Page 131, l. 3, for *treatiſes all of which* ; read *treatiſes, all of which*
132, l. 18, for *aſcertained* read *determined*
136, l. 5, note, for *Geometry* ; read *Aſtronomy*;
145, l. 1, note, for δ' ὁϖὲϱ read δ', ὁϖὲϱ
149, l. 12, for *plane*, read *plane*
170, l. laſt, for *ſame* read *fame*
175, l. laſt note, for *deperiſſe* read *deperijſſe*
176, the firſt note, from page 180, unneceſſary.

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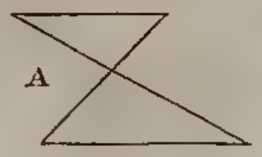
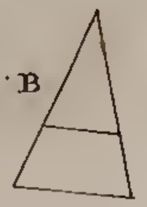




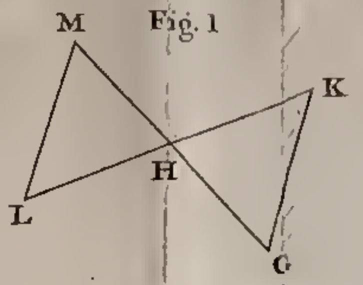
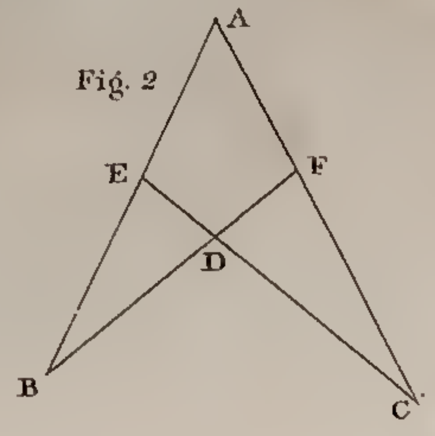


Fig. 1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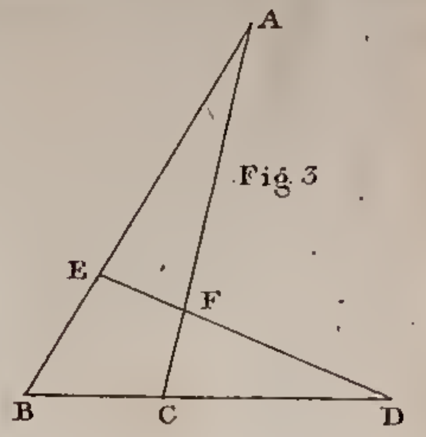
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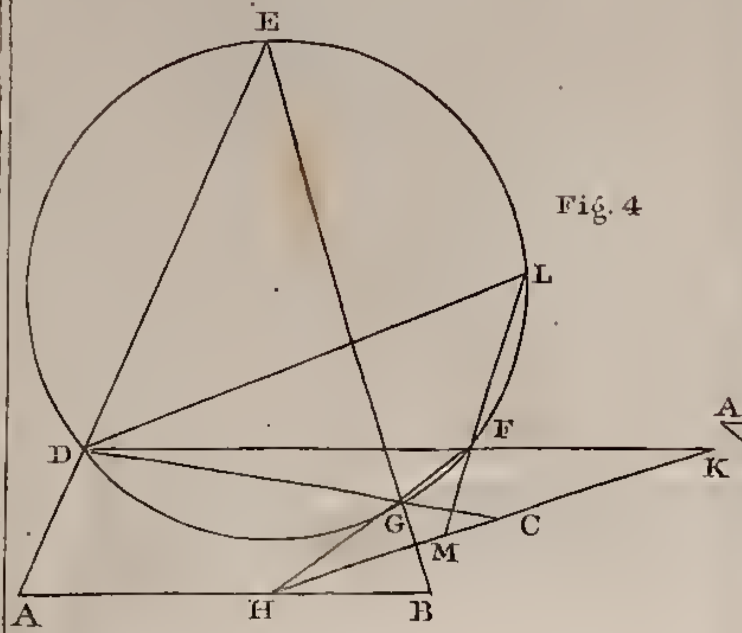
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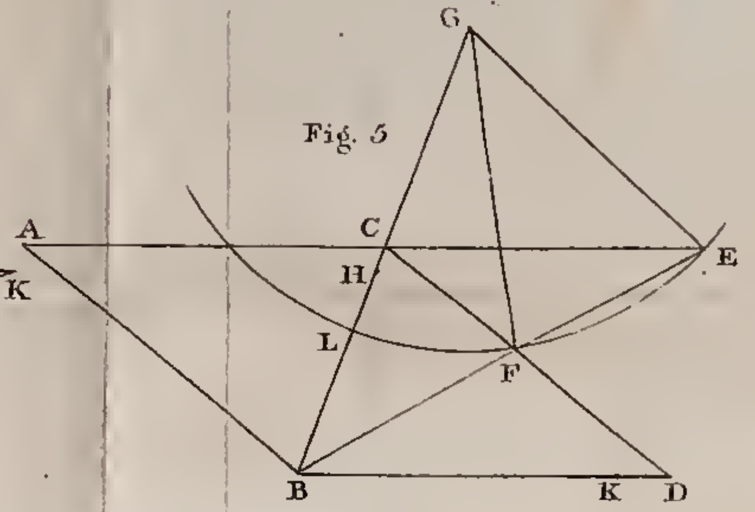
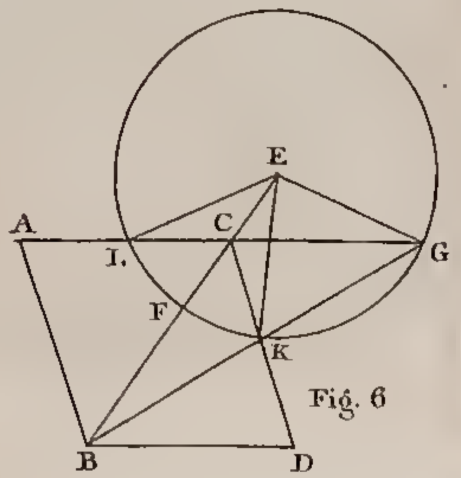


Fig. 5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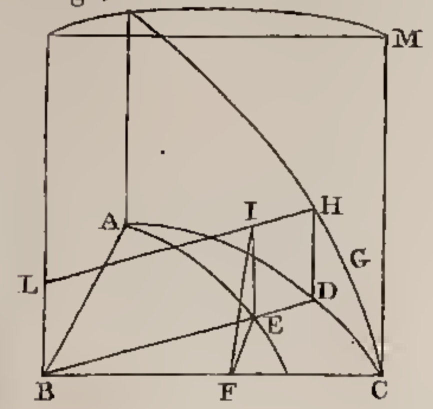
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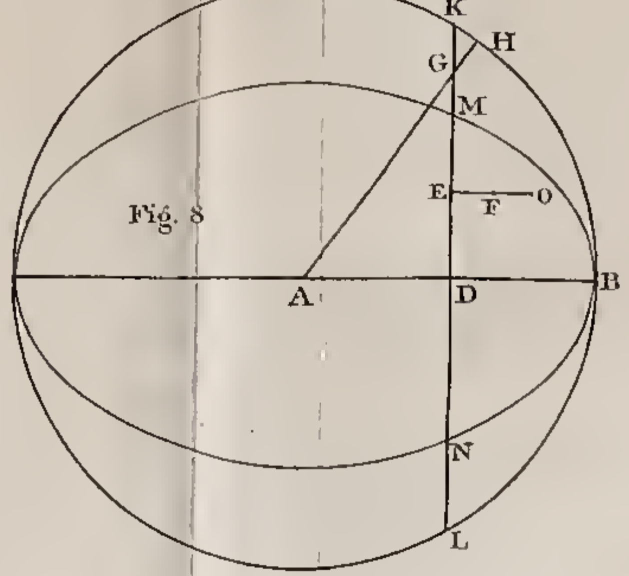
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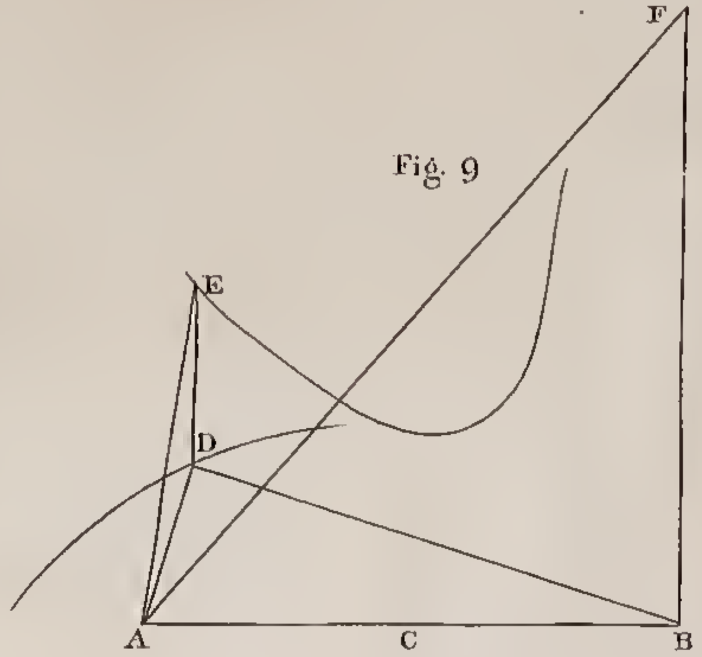
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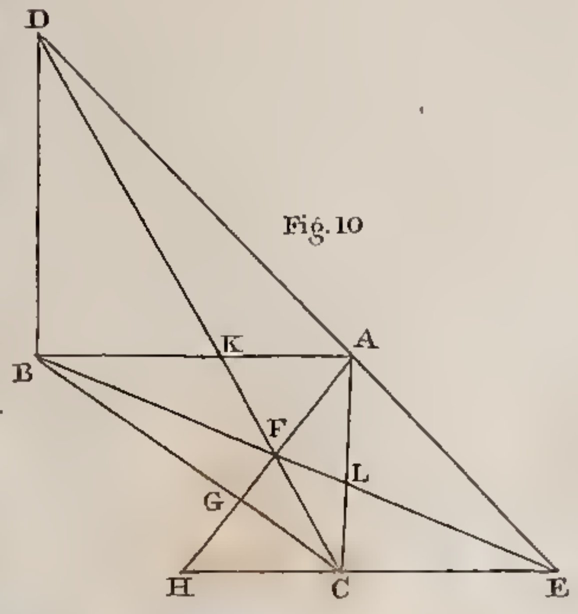
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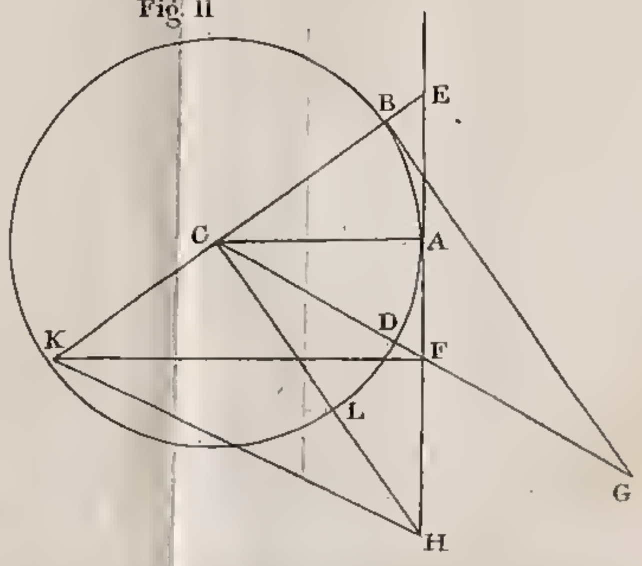
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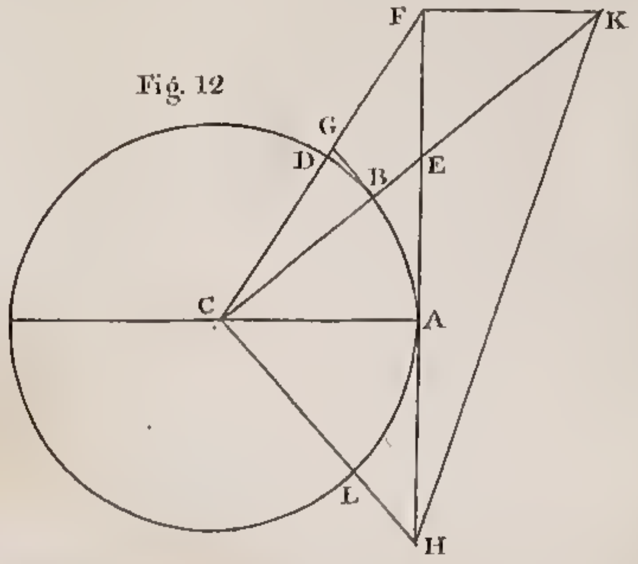
Fig. 12